AMSC/CMSC 460: Midterm 1

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

Problems: (Each problem = 10 points)

1. Consider the following matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 20 & 30 \\ 3 & 30 & 94 \end{pmatrix}$$

   (a) Find a Cholesky decomposition for A.

   (b) Explain how the Cholesky decomposition can be used to easily solve a linear system $Ax = b$.

2. Let $f(x) = x^3 - 2x - 3$.

   (a) Use the continuity of $f(x)$ to explain why $f(x)$ has at least one positive root.

   (b) Write Newton's method for finding a root of $f(x)$. Compute <u>two</u> iterations of the method starting from $x_0 = 2$.

   (c) Write the Secant method for finding a root of $f(x)$. Compute <u>one</u> iteration of the method starting from $x_0 = 0$ and $x_1 = 2$.

3. (a) Write the number 14.42 in base 2. (Compute the first 10 digits after the binary point).

   (b) Explain two possible approaches for representing the number -17 on a computer with an 8-bit word.

4. Let $f(x) = x^4 - 3x^2 + 3$.

   Let $x_0 = -1, x_1 = 1, x_2 = 2$, and let $y_j = f(x_j)$ for $j = 0, 1, 2$.

   (a) Write Newton's form for the interpolation polynomial that interpolates the data at the given points.

   (b) Write Lagrange's form for the interpolation polynomial that interpolates the data at the given points.

   (c) Assume that in addition to x_0, x_1, x_2, you are given one additional interpolation point $x_3 = 0$. Using the divided differences notation, write the term that should be added to the interpolation polynomial from part (a) in order to obtain a new polynomial that interpolates the data at all four points $x_0, \ldots, x_3$.