AMSC/CMSC 460: HW #5
Due: Tuesday 3/5/19 (in class)

> Please submit the solution to at least one problem in LaTeX.
> Problems 2–5 should be solved by hand.

1. Compute the infinity norm and the condition number in the infinity norm for the following two matrices: (you may use matlab to compute A^{-1})

$$A = \begin{pmatrix} -1.1 & 0.1 & 0.03 \\ 0.1 & 1.2 & 0.1 \\ 0.15 & -0.1 & 0.9 \end{pmatrix}, \qquad A = \begin{pmatrix} 2.1 & 2.15 & 1.2 \\ 2.1 & 2 & 1 \\ 1.9 & 2.1 & 1.1 \end{pmatrix}.$$

2. Find the Lagrange and Newton forms of the interpolating polynomials for the following sets of data. Write both polynomials in the form $a+bx+cx^2$ in order to verify that they are identical.

   (a)

x	1	2	3
$f(x)$	1	2	4

   (b)

x	2	0	-3
$f(x)$	4	7	-5

3. Write the unique cubic polynomial that interpolates the following values in Newton's form:

x	-4	2	1	5
$f(x)$	-3	-8	-6	1

4. Find a polynomial that interpolates the values of the function $f(x) = \exp(x)$ at $x_0 = -2$, $x_1 = 1$, and $x_2 = 2$ (a) in Lagrange's form, (b) in Newton's form. Check that these polynomials are identical.

5. The function $f(x) = x - 9^{-x}$ has a root in $[0, 1]$. Find the quadratic interpolation polynomial, $P_2(x)$, that interpolates the values of $f(x)$ at $x_0 = 0, x_1 = 0.25, x_2 = 1$. Find an approximation to the root of $f(x)$ by finding a root of $P_2(x)$.

6. *To be done in Matlab.*

   Load a function for computing the Lagrange interpolating polynomial from (using the "function" tab)
   https://www.mathworks.com/matlabcentral/fileexchange/13151-lagrange-interpolator-polynomial

   A sample code for using this routine is:

```
X = [1 2 3 4 5 6 7 8];
Y = [0 1 0 1 0 1 0 1];
[P,R,S] = lagrangepoly(X,Y);
xx = 0.5 : 0.01 : 8.5;
plot(xx,polyval(P,xx),X,Y,'or')
```

The sample code will interpolate the (X, Y) data, creating a polynomial P. The polynmoial is then plotted on top of the given data.

Use this routing to interpolate data from the following two functions:
(i) $f(x) = e^x$ on $[-3, 3]$,
(ii) $f(x) = \sin(\pi x)$ on $[-2, 2]$.

- First obtain the interpolation data, by using N equally spaced interpolation points. For each function you should repeat the problem 4 times, using$N = 3, 5, 10, 20$.
- Evaluate the interpolation polynomial at $10N$ equally spaced points. For example, if $N = 5$, you should sample the first function at the 5 points: -3, -1.5, 0, 1.5, 3. You should then set the vector xx to have 50 equally distributed points on the interval $[-3, 3]$. (It is ok to have 51 points if you prefer working with an odd number of points).
- Plot the graphs of the functions and the interpolation polynomials for the 4 different values of N.
- As a solution to this problem, you should only submit the graphs.