AMSC/CMSC 460: HW #7
Due: Thursday 3/28/19 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Read pages 41-44 in the lecture notes on Hermite interpolation. Find a quartic polynomial (written in Newton's form, i.e., using divided differences with repetitions) that takes these values: $p(0) = 1$, $p'(0) = -1$, $p(1) = -2$, $p'(1) = 2$, and $p(2) = 2$, This is problem is very similar to Example 3.19 in the notes. Check that the polynomial you obtained satisfies these interpolation conditions.

2. A natural cubic spline is defined as a cubic spline for which the second derivative is zero at the first and last knots. Find a natural cubic spline function whose knots are $-3, 0, 1$ and that takes these values

x	-3	0	1
y	1	-2	4

3. Determine all the values of a, b, c, d, e for which the following function is a cubic spline

$$f(x) = \begin{cases} a(x-2)^2 + b(x-1)^3, & x \in (-\infty, 1], \\ c(x-2)^2, & x \in [1, 3], \\ d(x-2)^2 + e(x-3)^3, & x \in [3, \infty). \end{cases}$$

Next, determine the values of the parameters so that the cubic spline interpolates this table

x	0	1	4
y	3	-1	5

4. Use Matlab's built-in *spline* routine to plot a cubic spline function that interpolates the following 11 points:

$$x_i = i/10, \quad y_i = e^{x_i}, \quad i = 0, \ldots 10.$$

If you have access to Matlab's spline toolbox, use the *csape* routine to plot the spline function that interpolates this exponential data with different boundary conditions (try not-a-knot, periodic, etc.). See `https://www.mathworks.com/help/curvefit/csape.html`