Math412 – Midterm 1

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Instructions:

(i) Read each problem carefully.

(ii) Write clearly and show all your work in your notebook.

(iii) You may NOT use calculators, books, or notes.

Good luck!

Problem 1. (25 points)

(a) Is a union of a collection of closed sets closed? Explain.

(b) Let $A \subset \mathbb{R}^2$ be the following set:

$$A = \{(x, y) : \ y \leq e^x \text{ and } y \leq e^{-x}\}.$$

(i) Sketch A. (ii) Is the set open, closed or neither? (iii) Describe $A^0, \bar{A}, \partial A$.

Problem 2. (25 points)

(a) Without using the words "closed" and "bounded", define a compact set in $\mathbb{R}^n$.

(b) Prove that a closed and bounded set in $\mathbb{R}^n$ must be compact.

Problem 3. (25 points)

(a) Prove that the following set $A \subset \mathbb{R}^2$ is open

$$A = \{(x, y) : \ x > 0, y > 0\}.$$

(b) Let $A \subset \mathbb{R}^n$ be compact. Let $f : A \to \mathbb{R}^m$ be continuous. Prove that $f(A)$ is compact.

Problem 4. (25 points)

a) What are the partial derivatives f_x and f_y of the following function at $(0, 0)$

$$f(x, y) = 2 + x + |x|y.$$

b) Does $f(x, y)$ have a linear approximation at $(0, 0)$? If yes, what is it?