Math412 – Midterm 2
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April 5, 2011

Instructions:
(i) Read each problem carefully.
(ii) Write clearly and show all your work in your notebook.
(iii) You may NOT use calculators, books, or notes.

Good luck!

Problem 1. (20 points)

Assume that $a, b, c \in \mathbb{R}$. Compute $\|A\|_\infty$ and $\|A\|_1$ for $A = \begin{pmatrix} a & b & -c \\ -a & -b & -c \\ 2a & -2b & 2c \end{pmatrix}$.

Problem 2. (15 points)

Compute the Jacobian of the following mapping, $\vec{f} : \mathbb{R}^2 \to \mathbb{R}^3$, $\quad \vec{f}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \log(\sin(x-2y)) \\ \\ x + e^{-(2x+y)} \\ \\ \sin(x) \end{pmatrix}$.

Problem 3. (15 points)

Let $g : \mathbb{R} \to \mathbb{R}^2$ with $g(t) = (g_1(t), g_2(t))$. Let $\vec{f}(x_1, x_2) = \begin{pmatrix} 2x_1x_2 \\ x_1 - 2x_2 \end{pmatrix}$. Use the chain rule to compute $\frac{d}{dt}(f \circ g)(t)$. Express the solution in terms of g_1, g_2, g_1', g_2'.

Problem 4. (25 points)

Prove that the mapping $\vec{f} : D \to \mathbb{R}^2$ defined as

$$\vec{f}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\dfrac{y}{2} \\ \\ \dfrac{x}{3} \end{pmatrix},$$

has a unique fixed point in the domain $D = \{(x, y) : \ x^2 + y^2 \leq 1\}$.

Problem 5. (25 points)

Consider the mapping $\vec{f} : \mathbb{R}^2 \to \mathbb{R}^2$, defined as $\vec{f}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \sin(xy) \\ \\ e^{1-x} - \left(\frac{y}{\pi}\right)^3 \end{pmatrix}$. Explain why $\vec{f}$ has a linear approximation at $\begin{pmatrix} 1 \\ \pi \end{pmatrix}$ and find it.