Homework 1. Selected Solutions, 9/12/03.

Ch. 2, # 34, p. 74. The most natural sample space S is the set of unordered 6-tuples, i.e., the set of 6-element subsets of the list of 45 workers (labelled $1, \ldots, 45$, of whom we can imagine the first 20 work on the day shift, numbers 21 to 35 work on the swing shift, and numbers 36 to 45 work on the night shift.) Since sampling is done fairly, all $\binom{45}{6} = (45{\cdot}44{\cdot}43{\cdot}42{\cdot}41{\cdot}40)/6! =$ 8145060 outcome-selections are equiprobable.

(a) All workers come from the day shift in $\binom{20}{6} = 38760$ selected subsets: so this event occurs with probability $\binom{20}{6}/\binom{45}{6} = 0.004759$.

(b) Similarly, there are respectively $\binom{15}{6}$ and $\binom{10}{6}$ ways to choose subsets of workers all from the swing shift or all from the night shift, and the overall probability of choosing all 6 from the same shift is $(\binom{20}{6}+\binom{15}{6}+\binom{10}{6})/\binom{45}{6} =$ 0.005399.

(c) The event that at least 2 different shifts are represented among selected workers is the complement of the event in (b), so the probability is $1 - 0.005399 = 0.9946$.

(d) Let A, B, C respectively be the events that the day, swing, and night shifts are unrepresented among the selected workers. The probability we are asked for is $P(A \cup B \cup C)$, and the probabilities of the events A, B, C and their overlaps are easily seen to be:

$$P(A) = \frac{\binom{25}{6}}{\binom{45}{6}} \quad , \quad P(B) = \frac{\binom{30}{6}}{\binom{45}{6}} \quad , \quad P(C) = \frac{\binom{35}{6}}{\binom{45}{6}}$$

$$P(A \cap B) = \frac{\binom{10}{6}}{\binom{45}{6}} \;, \quad P(A \cap C) = \frac{\binom{15}{6}}{\binom{45}{6}} \;, \quad P(B \cap C) = \frac{\binom{20}{6}}{\binom{45}{6}} \;, \quad P(A \cap B \cap C) = 0$$

Therefore, substituting into the formula in the middle of p. 63 of Devore,

$$P(A \cup B \cup C) \;=\; \frac{1}{\binom{45}{6}} \left(\binom{25}{6} + \binom{30}{6} + \binom{35}{6} - \binom{10}{6} - \binom{15}{6} - \binom{20}{6} \right) = 0.2885$$

Ch. 2, # 41: (a) $P(\geq 1 \; female) = 1 - (\binom{4}{0}\binom{4}{3}/\binom{8}{3})$. (b) $P(met\ with\ all\ in\ 5\ appts.) = \binom{4}{4}\binom{4}{1}/\binom{8}{5}$. (c) The sample space for the new ordering consists of the $8!$ orderings, and the chance that a second ordering randomly selected agrees with the previous one is $1/8!$.

Homework 2. Selected Solutions, 9/24/03.

Ch. 2, # 39: Think of the 15 phones as 15 numbered cards, of which the first 5 are C (Cell-phone), the next 5 are H (Cordless), and the last 5 are L (Corded). The cards are shuffled and 10 dealt. (a) asks for the probability (among the $\binom{15}{5} = \binom{15}{10}$ equally likely sets of 5 cards **not dealt**) that all 5 cards are not C or H. Answer: $\binom{10}{5}/\binom{15}{5} = 252/3003 = .0839$. (b) Let B_1, B_2, B_3 respectively be the events that no C cards are among the last 5, that no H cards are among them, and that no L cards are. We found $P(B_1)$ in (a), and $P(B_2)$, $P(B_3)$ have exactly the same value. Note also that all of the pairwise overlaps of these events have the same probability $P(B_1 \cap B_2) = \binom{5}{5}/\binom{15}{5} = 1/3003$, while the triple-overlap event $B_1\mathcal{B}_\in \cap \mathcal{B}_\ni$ is empty (thus has probability 0) by it definition. The desired probability for this problem-part, by my legalistic reading of the problem, is $P\Big((B_1 \cap B_2 \cap B_3^c) \cup (B_1 \cap B_2^c \cap B_3) \cup (B_1^c \cap B_2 \cap B_3)\Big) = P(B_1) + P(B_2) + P(B_3) - 2P(B_1 \cap B_2) - 2P(B_1 \cap B_3) - 2P(B_2 \cap B_3) = (3\binom{10}{5} - 6)/\binom{15}{5} = .24975$. (Look at the Venn Diagram to confirm this !!)

Ch. 2, # 42: As hinted in class, the sample space consists of 6! equiprobable left-to-right orderings of the 6 people $H_1, W_1, H_2, W_2, H_3, W_3$. (a) $P(H_1, W_1 \text{ at far left}) = 2(4!)/6! = 1/15$. (b) For $P(H_1$ next to $W_1)$, first count positions for the leftmost of H_1, W_1 to sit, then the choices of which sits at left, then the other 4 people's positions, getting $5 \cdot 2 \cdot 4!/6! = 1/3$. (c) If we let A_i for $i = 1, 2, 3$ denote the event that H_i, W_i sit together, then we have found $P(A_i) = 1/3$. Next must find $P(A_1 \cap A_2) = (2 \cdot 3 \cdot 2^3 + 2 \cdot 2 \cdot 2^3 + 1 \cdot 2 \cdot 2^3)/6! = .1333$. (The three terms in the numerator correspond respectively to the cases where couple 1 sits at one end of the row, one seat in from the end, or in the middle). Also $P(A_1 \cap A_2 \cap A_3) = 3!2^3/6! = 0.0667$. Putting it all together, our answer is: $P(A_1 \cup A_2 \cup A_3) = 3P(A_1) - 3P(A_1 \cap A_2) + P(A_1 \cap A_2 \cap A_3) = 2/3$.

Ch. 2, # 78: If A_i is the event that component i works, then these events for $i = 1, 2, 3, 4$ are mutually independent, each with probability .9. The probability the system works is $1 - P(A_1^c \cap A_2^c \cap (A_3 \cap A_4)^c) = 1 - (.1)^2(1 - .9^2) = .0019$.

HW2 solutions continued next page.

Ch. 3, # 20: The 16 outcomes indicated in the hint are (a, b) where a denotes the day of arrival for magazine 1, b the day for magazine 2. Then

$$p_Y(0) = P(\{(W, W)\}) = .3^2 = .09$$

$$p_Y(1) = P(\{(Th, Th),\ (W, Th),\ (Th, W)\}) = .4^2 + 2(.4)(.3) = .4$$

$$p_Y(2) = P(\{(F, F),\ (W, F),\ (Th, F),\ (F, W),\ (F, Th)\}) = .2^2{+}2(.2)(.7) = .32$$

and we conclude that the final nonzero pmf value is $p_Y(3) = 1 - p_Y(0) - p_Y(1) - p_Y(2) = 1 - .81 = .19$.

Homework 3. Selected Solutions, 10/1/03.

Ch. 2, # 62: Let B denote the event that the (next) buyer's video camera is of the basic model (so that B^c is the event that it is deluxe); and let E be the event that the purchaser buys an extended warranty. The information given in the problem says directly that:

$$P(B) = 0.4\ \ ,\quad P(E \,|\, B) = 0.3\ \ ,\quad P(E \,|\, B^c) = 0.5)$$

and the desired probability is $P(B|E) = 2/7$, calculated as $P(B \cap E)/\{P(B \cap E) + P(B^c \cap E)\} = P(B)P(E|B)/ \{P(B)P(E|B) + P(B^c)P(E|B^c)\} = (0.4)(0.3)/\{(0.4)(0.3) + (0.6)(0.5)\}$.

Ch. 2, # 83: (a) Probability of a flaw by the second fixation $=$ $P(\text{1st is flaw}) + P(\text{1st is not flaw and 2nd is flaw}) = p + (1-p)p = (2-p)p$. (b) Probability detected by the end of the n'th fixation is the probability of at least one success in n Binomial(1,p) trials $= 1-(1-p)^n$. (c) Ans. in (b) for $n = 3$) is $1-(1-p)^3$. (d) Let F be the event of a flaw, then $P(F) = 0.1$ and the answer to (c) is $P(\text{Pass}^c|F)$, so $P(\text{Pass}|F) = (1-p)^3$, and we can assume $P(\text{Pass}|F) = 1$, so that $P(\text{Pass}) = 0.1(1-p)^3 + 0.9$. (e) Now we want, with $p = 0.5$, $P(F \,|\, \text{Pass}) = 0.1(1-p)^3)/\{.9 + 0.1(1-p)^3\} = (.1/8)/(.9 + 0.1/8) = 0.01370$.

HW3 solutions continued next page.

Ch. 3, # 62: (a). Probability of the event that there are at most 4 successes in a Binom(6,0.8) experiment is $B(4,6,.8) = 1 - \binom{6}{5}(.8)^5(.2) - \binom{6}{6}(.8)^6 = 0.34464$. (b) 'Available places' are 0 if number X arriving (a Binom(6,.8) r.v.) is 4 or more, and otherwise is 4-X. So the expectation is $P(X \geq 4) * 0 + P(X = 3) * 1 + P(X = 2) * 2 + P(X = 1) * 3 + P(X = 0) * 4 = 0.11754$. (c) This time, the number R of reservations is random (where before it was assumed to be 6). Once R=r is fixed, the number of arriving passengers is Binom(r,.8), and the number actually on the trip is the smaller of the number arriving and 4. So for example, $P(X = 4) =$

$$P(R = 4)\binom{4}{4}.8^4 + P(R = 5)(1 - B(3,5,.8)) + P(R = 6)(1 - B(3,6,.8))$$

or $p_X(4) = 0.66355$. The other probabilities $P(X = k)$ for $k = 0, \ldots, 3$ can be calculated using the formula $\sum_{r=3}^{6} \frac{r-2}{10} \binom{r}{k} .8^k \, .2^{r-k}$, giving the results $p_X(0) = 0.00124$, $p_X(1) = 0.01725$, $p_X(2) = 0.09062$, $p_X(3) = 0.22733$.

Ch. 3, # 83: Sorry, I should not have asked you a 'Poisson process' question. (a) If the rate (= expected number) is 4 per hour, then for a 2-hour period the rate is $\lambda = 8$. For Poisson(8), the probability of a value exactly 10 is $e^{-8}8^10/10! = 0.09926$. (b) Now for the missed 30-minute period, the rate (= expected number) is $0.5(4) = 2$, and the Poisson(2) probability of 0 is $e^{-2} = 0.13534$. (c) For Poisson(2) the expectation is 2.

Homework 4. Selected Solutions, 10/20/03.

Ch. 3, # 65: (c). We know that the mean and standard deviation for a Hypergeometric with $N = 12$, $D = 7$, $n = 6$ are respectively $\mu = (7)(6)/12 = 3.5$, $\sigma = \sqrt{\mu \cdot \frac{6}{11}(1 - \frac{7}{12})} = 0.892$. Therefore the probability that the hypergeometric variable differs by more than one standard deviation from the mean is $P(X < 3.5 - 0.892) + P(X > 3.5 + 0.892) = P(X \leq 2) + P(X \geq 5) = 1 - P(X = 3 \text{ or } 4) = 0.242$. (d) In this problem, we are sampling without replacement from a rather large population, which means that the answer will be approximately the same *with replacement*, i.e. as a Binomial$(n, D/N)$ probability. The exact hypergeometric answer is 0.9982, while the Binomial probability is $B(5, 15, 0.1) = 0.9977$.

Ch. 3, # 71: (a). This is the same as the probability as that there is exactly one female among the first $x + 1$ children times 0.5 (the probability that the x'th child is female), or $(x+1)\cdot 2^{-x-2}$. (b). This is the answer in (a) with $x=2$, or $3/16$. (c). This is the probability of FF, or MFF, or FMF, or MFMF, or MMFF, or FMMF = $1/4 + 2/8 + 3/16 = 11/16$. (d). Expected number of children until the first female is 2 (including that female). Since this must happen twice in succession, the expected number of children overall is 4, which makes the expected number of males 2.

Ch. 3, # 31: (b). $E(25X-8.5) = 25E(X)-8.5 = 25(16.38)-8.5 = 401$. (c). $\text{Var}(25X-8.5) = 25^2\,\text{Var}(X) = 2496$.

Ch. 4, # 6: (e). $1-0.75\int_{2.5}^{3.5}(1-(x-3)^2)dx = 1-0.75\int_{-.5}^{.5}(1-z^2)dz = 0.3125$.

Ch. 4, # 11: (h). $\int_0^2 x^2\,(x/2)\,dx = 2^4/8 = 2$.

Ch. 4, # 32: (c). Solve $\Phi((x-.3)/.06) = 0.95$ by solving $(x-.3)/.06 = 1.645$, or $x = (.06)1.645 + .3 = 0.3987$.

Ch. 4, # 50: (b). $0.9836 = B(29,200,0.1) \approx \Phi((29-20)/\sqrt{18}) = 0.9831$, or, with continuity correction, $\approx \Phi((29.5-20)/\sqrt{18}) = 0.9874$. (c) $0.8066 = B(25,200,0.1) - B(14,200,0.1) \approx \Phi((25-20)/\sqrt{18}) - \Phi((15-20)/\sqrt{18}) = 2\Phi(5/\sqrt{18}) - 1 = 0.7614$.

HOMEWORK 5. SELECTED SOLUTIONS, 10/30/03.

Ch. 4, #24: $EX^m = \int_\theta^\infty k\,\theta^k\,x^m\,x^{-k-1}\,dx = k\theta^k\,\theta^{-k+m}/(k-m) = k\theta^m/(k-m)$ if $m<k$, but equals $+\infty$ if $m \ge k$.

Ch. 4, #35: Generally, the p'th quantile of a normal is the solution x of $\Phi((x-\mu)/\sigma) = p$, or $x = \mu + \sigma\,z_{1-p}$ where z_α denotes the entry in stanfdard normal tables where $\Phi(z_{1-p}) = p$. (b). Here we have $\Phi(-1.555) = .06$, so the 6'th percentile is $30 + 5(-1.555) = 22.225$.

Ch. 4, #42: (b) We want c so that $\Phi((70+c-70)/3) - \Phi((70-c-70)/3) = 0.95$, i.e., $\Phi(c/3) - \Phi(-c/3) = .95$. There is a general identity for normal distribution function values that $\Phi(-x) = 1 - \Phi(x)$, which implies here that $\Phi(-c/3) = 1 - \Phi(c/3)$, and $\Phi(c/3) - \Phi(-c/3) = 2\Phi(c/3) - 1$. We set this equal to 0.95, implying that $\Phi(c/3) = 1.95/2 = 0.975$, and from the normal table this holds when $c/3 = 1.960$, so that $c = 5.88$. (c) Each specimen has probability exactly 0.794 of falling in the range $(67,75)$ found in (a). Thus, out of 10 independent specimens, the number falling in the range is $Binom(10, .794)$, and the expected number falling in the range is 7.94. (d) The probability that each specimen has hardness less than 73.84 is $p = \Phi((73.84-70)/3) = \Phi(1.28) = .9$. So out of 10 independent specimens, the probability that 8 or fewer have hardness less than 73.84 is $B(8, 10, .9) = 0.2639$.

Ch. 4, #64: The equation we are solving is $F(x) = 1 - e^{-\lambda x} = p$, which gives the p'th quantile as solution: $x = -\log(1-p)/\lambda$. Thus all quantiles are proportional to the mean, which was $1/\lambda$, and in particular the median is equal to $\log(2)/\lambda$.

Ch. 4, #72: By assumption, $\log X \sim \mathcal{N}(4.5, 0.8^2)$. (b) So $P(X \leq 100) = P(\log X \leq \log 100) = \Phi((\log 100 - 4.5)/0.8) = 0.552$.

Ch. 5, #7: (b) $P(X \leq 1, Y \leq 1) = .025 + .015 + .05 + .03 = .12$. (d) We are asked to find $P(X + 3Y \geq 6) = P(Y = 2) + P(Y = 1, X \geq 3) = 0.38$. Note that by summing over rows and columns respectively, we find that $p_Y(0) = .5$, $p_Y(1) = .3$, $p_Y(2) = .2$, and

$$p_X(0) = .05,\ p_X(1) = .1,\ p_X(2) = .25,\ p_X(3) = .3,\ p_X(4) = .2,\ p_X(5) = .1$$

From this, we check easily that X, Y are **are** independent, since the three columns of the table do stand in the proportions 5 : 3 : 2.

Ch. 5, #26: (*correlation part*) Since we found the variables X, Y in problem 7 to be independent, their correlation is 0. The ain part of the problem is to find $E(3X + 10Y) = 3E(X) + 10E(Y)$, which can be found either directly from the Table or from the marginal probability mass functions given in #7: $EX = 1(.1) + 2(.25) + 3(.3) + 4(.2) + 5(.1) = 2.8$, $EY = 1(.3) + 2(.2) = .7$.

TRAN.1 We did this one in class: the transformation g satisfies the equation $g^{-1}(y) = (y-12)/5$ for $12 \leq y \leq 17$, or $g(x) = 5x + 12$.

TRAN.2 $f_V(v) = e^{-v}$, and $F_V(v) = 1 - e^{-v}$ for $v > 0$; also $g(v) = 3v^2 + 1$, so $g^{-1}(y) = ((y-1)/3)^{1/2}$. Thus if $Y = 3V^2 + 1$, then $F_Y(y) = \exp\Big(-((y-1)/3)^{1/2}\Big)$, for $y > 1$, and

$$f_Y(y) = \exp\Big(-((y-1)/3)^{1/2}\Big)(1/(2\sqrt{3}))\,(y-1)^{-1/2}$$

also for $y > 1$.

SIM.1 We solve $u = F_W(w) = 1 - e^{-2w^5}$, to find $w = (-\log(1-u)/2)^{1/5}$. Thus the simulation algorithm is: $W_i = (-\log(1-U_i)/2)^{1/5}$.

Homework 6. Selected Solutions, 11/5/03.

Ch. 5, #22: (b). $E(max(X,Y)) = 15 \cdot P(Y = 15) + 10 \cdot P(Y = 10) + 5 \cdot (P(X \le 5, Y \le 5) - P(X = Y = 0)) + 0 \cdot P(X = Y = 0) = 15(.21) + 10(.36) + 5(.25) + 0 = 3.15 + 3.6 + 1.25 = 8$. *For correlation*:

$$p_X(0) = .2, p_X(5) = .49, p_X(10) = .31, EX = 5.55, EX^2 = 43.25, Var(X) = 12.45$$

$$p_Y(0) = .07, p_Y(5) = .36, p_Y(10) = .36, p_Y(15) = .21, EY = 8.55, \mathrm{Var}E(Y) = 19.15$$

and then, after directly calculating $E(XY) = 44.25$, $\mathrm{Cov}(X,Y) = -3.2025$,

$$Cor(X,Y) = (44.25 - (5.55)(8.55))/\sqrt{(12.45)(19.15)} = -0.207$$

Ch. 5, #37: (b). s^2 denotes the sample variance $\frac{1}{n-1}\sum_{i=1}^n (X_i - \overline{X})^2$, and with $n = 2$ we find this $=$ $(X_1 - X_2)^2/2$. The sampling distribution is discrete with probability mass 0.38 at $s^2 = 0$, mass 0.3 at $s^2 = (25)^2/2 = 312.5$, 0.2 at $(15)^2/2 = 112.5$, and mass 0.12 at $s^2 = (40)^2/2 = 800$. *The expectation is* $212.25 = \sigma^2$.

Ch. 5, #48: (b). $\Phi((50.25-49.8)/(1/\sqrt{100})) - \Phi((49.75-49.8)/(1/\sqrt{100})) = 1.000 - 0.309 = 0.691$.

Ch. 5, #56: (a) $\Phi((70-50)/\sqrt{50}) - \Phi((35-50)/\sqrt{50}) = 0.981$. (b). $\Phi((275-250)/\sqrt{250}) - \Phi((225-250)/\sqrt{250}) = 0.886$.

Sim.3: (a) N is the total count of 'successes' in 1000 independent trials, where success on the i'th trial means that $U_i \ge 0.6$, and this is an event with probability $p = 0.4$. Thus we recognize that $N \sim \mathrm{Binom}(1000, 0.4)$. (b) The law of large numbers says that $N/1000$, the relative frequency of success, should be close with high probability to the probability of success, or to $p = 0.4$ (c) Again the law of large numbers says that A, which is the average of $n = 1000$ independent and identically distributed variables $X_i = \exp(-U_i)$, should be close with high probability to the expectation of any one of them, i.e. to $E(U_1) = \int_0^1 e^{-u}\,du = 1 - e^{-1} = 0.632$. (d) The r.v. $e^{-U_i} = g(U_i)$, with $g(u) = e^{-u}$ and $g^{-1}(x) = -\ln(x)$, has distribution function $1 - F_U(g^{-1}(x)) = 1 - g^{-1}(x) = 1 + \ln(x)$, for $e^{-1} \le x \le 1$. Here we use the formula $F_X(x) = P(e^{-U} \le x) = P(U \ge -\ln(x)) = 1 - F_U(-\ln(x))$, which differs from the one we usually use because g is here monotone *de*creasing.

HOMEWORK 7. SELECTED SOLUTIONS, 11/24/03.

Ch. 5, #56: (a)-(b) $E(27X_1+125X_2+512X_3) = 27\mu_1+125\mu_2+512\mu_3$ whether or not the randoim variables are independent. But variances of (weighted) sums of variables are equal to sums of variances without covariance cross-terms only if we assume independence (or more generally, 0 covariances), in which case $Var(27X_1+125X_2+512X_3) = (27)^2\sigma_1^2+(125)^2\sigma_2^2+(512)^2\sigma_3^2$.

Ch. 5, #60: $Y \sim \mathcal{N}(\frac{1}{2}\cdot 2\cdot 20-\frac{1}{3}\cdot 3\cdot 21,\ \frac{1}{2^2}\cdot 2(4)+\frac{1}{3}^2\cdot 3(3.5)) \sim \mathcal{N}(-1,\ \frac{19}{6})$, and we calculate probabilities for Y using this distribution.

Ch. 6, #3: (a) $\overline{X} = 1.348$. The same estimator applies in (b), since mean and median are the same for the normal distribution. (c) $\hat{\mu} + \Phi^{-1}(.9)\hat{\sigma} = \overline{X} + 1.282\, s = 1.782$. (d) $\Phi((.15 - \overline{X})/s) = \Phi((1.5 - 1.348)/0.339) = 0.673$. (e) Standard error for $\overline{X}$ is $\sigma/\sqrt{16}$, which is estimated by $s/4 = 0.085$.

Ch. 6, #6: (a) Take sample mean and standard deviation of the natural logs of the observations to find $\hat{\mu},\ \hat{\sigma}^2$, since the log of a lognormal($\mu,\ \sigma^2$) is $\mathcal{N}(\mu, \sigma^2)$. The answers are: $\hat{\mu} = 5.102, \hat{\sigma} = 0.496$. (b) By formula in Sec. 4.5, $E(X) = \exp(\mu + \frac{1}{2}\sigma^2)$ is estimated by $\exp(5.102 + 0.496^2/2) = 185.9$.

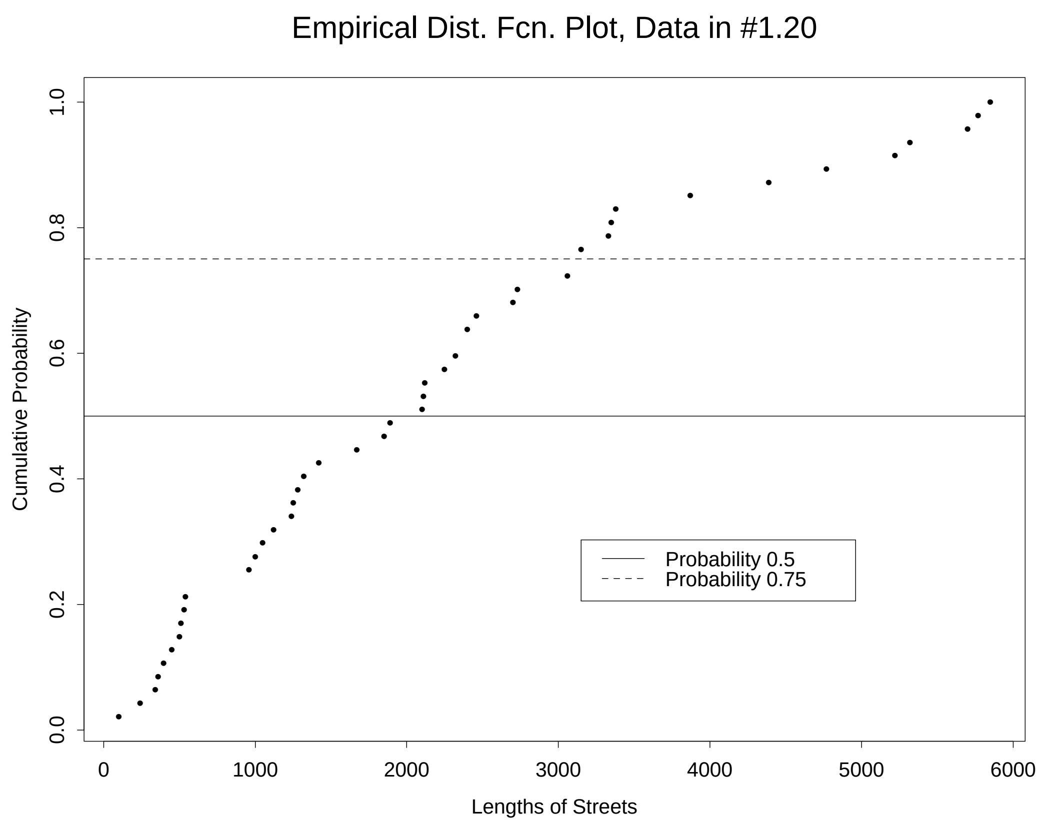
Ch. 6, #9: (a) Enough information is given for us to calculate the unbiased estimator $\hat{\lambda} = \overline{X} = \sum_{k=0}^{7} k \cdot f_k / \sum_{k=0}^{7} f_k = 317/150 = 2.113$. (b) Since $\sigma^2 = \lambda$ for Poisson, the estimated standard error is $\hat{\lambda}^{1/2}/\sqrt{n} = \sqrt{2.113/150} = 0.119$.

Ch. 1, #34(b): Mean and median will differ a lot when there are a few very large observations, as here.

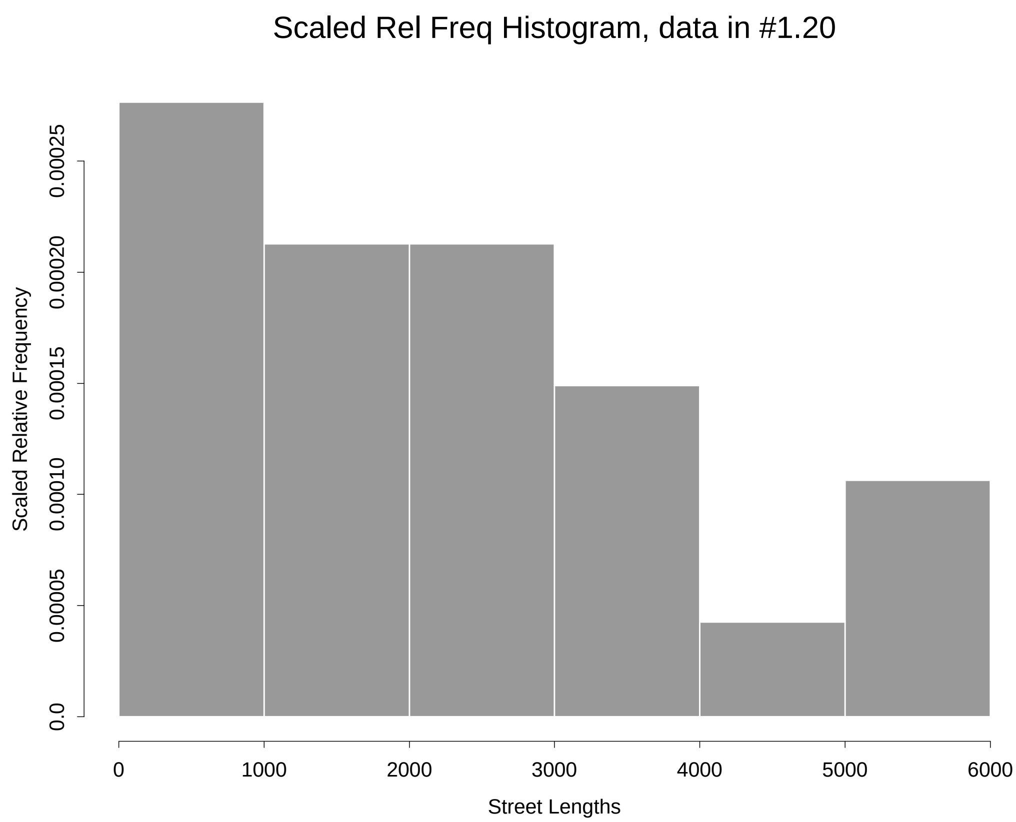
Ch. 1, #39(b): The largest observation (1.394) could be brought all the way down to 1.011 without changing the median $= (1.007 + 1.011)/2$.

Ch. 7, #14: (a) $\overline{x} \pm 1.96 s/\sqrt{n} = (88.54,\ 89.66)$. (b) Here we are asked to find n so large that $1.96(4)/\sqrt{n} \le 0.5$, or $n \ge (7.84/0.5)^2 = 245.9$.

Ch. 7, #20: $0.15 \pm 2.576\sqrt{.15(.85)/4722} = (0.137,\ 0.163)$.



Extra Problem on empirical distribution function and scaled relative frequency histogram: See the accompanying pictures. In the first picture, we plot against the ordered increasing distinct data values $X_{(k)}$ $1/47$ times the the number of data values $\leq X_{(k)}$. The sample median and upper quartile are respectively 2100 and 3150, which can be read off reasonably well from the empirical distribution function because we plot the horizontal probability line at 0.5 (solid) and at 0.75 (dashed). Next, for the histogram, we count respectively 13, 10, 10, 7, 2, 5 in the successive class intervals, and the histogram picture is attached. The vertical scaled unit (per count) is $1/47000$.



Extra CLT Problem: As done in class, we first must use the known form of mean $\mu = 1/2$ and variance $\sigma^2 = 1/4$ for $\mathit{Expon}(2)$ r.v. to find the mean and variance of $\sum_{i=1}^{60} X_i$ are respectively $60(1/2) = 30$ and $60(1/4) = 15$. Therefore

$$P(26 \leq X_1 + \cdots + X_{60} \leq 36) \approx \Phi\Big(\frac{36-30}{\sqrt{15}}\Big) - \Phi\Big(\frac{26-30}{\sqrt{15}}\Big) = 0.7884$$

Homework 8. Selected Solutions, 12/0/03.

Ch. 7, #21. One-sided lower-bounding interval for binomial proportion. The simpler form of interval is $(133/539 - 1.645\sqrt{\frac{(133)(406)}{539^3}},\ \infty) = (0.216, \infty)$. The more complicated and accurate form involving quadratic formula is:

$$\Big(\Big(\frac{133}{539} + \frac{1.645^2}{2\cdot 539} - 1.645\sqrt{\frac{133\cdot 406}{539^3} + \frac{1.645^2}{4\cdot 539^2}}\Big)/(1+1.645^2/539), \infty\Big)$$

from which the lower bound is computed as 0.2175.

Ch. 7, #25. (a) Formula is: $4z^2_{\alpha/2}p(1-p)/(0.1)^2$ with $p = 1/2$ to give largest value of required sample size, or $1.96^2/.01 = 385$ (rounding up). (b) Now use same formula but with $p = 2/3$ to get $4*(1.96^2)*100*2/9 = 342$.

Ch. 7, #34. (a) $(8.48 - (1.771)\,(.79)/\sqrt{14} = (8.106, \infty)$,
(b) $8.48 - (1.771)\,(.79)\,\sqrt{1+1/14} = 7.032$.

Ch. 7, #38. (a) The point prediction is simply $\overline{X} = .0635$, and the requested 'precision' is the estimated standard error of $.0065\sqrt{1+1/14} = 0.0067$. (b) The 95% prediction interval is then $.0635 \pm 2.064(.0067) = (.0497, .0773)$.

Ch. 7, #48. $188 \pm t_{8,.01}\, 7.2/\sqrt{9} = (163.84, 212.16)$.

Ch. 7, #52. Must assume normally distributed observations . (a) $.214 \pm 1.753(.036)/\sqrt{16} = (0.198, 0.2230)$. (b) This involves the χ^2 table: we did not study this topic, sorry, so this part will not be graded. (c) 95% prediction interval $= .214 \pm 2.13(.036)\sqrt{1+1/16} = (0.135, 0.293)$.