Stat 400 In-Class Test 1 Fall, 1996 10/2/96

Instructions: Do each of the following problems; each is worth 20 points. You may use calculators, a notebook sheet of notes, and the tables in the back of your books. You need not give reduced numerical answers: numerical expressions which are readily evaluated on a calculator are good enough, unless a decimal answer is specifically requested. (But do not leave the answer in the form of an integral or abstract symbol). Justify your steps wherever you can: I will be much more generous with partial credit wherever you explain clearly what you are doing.

(1). Suppose that computer-account passwords consist of strings of 6 characters (like **TT5bBW**) , where each character is either a number $0 \ldots 9$, an upper-case letter $A \ldots Z$, or a lower-case letter $a \ldots z$ (62 possible characters in all). What is the probability that a randomly selected password

(i) contains at least 2 upper-case letters ?

(ii) contains 6 different characters ? (Note that the upper and lower case of the same letter, such as a and A, count as different characters.)

(2). In a certain population of 1,000 people, exactly 55 happen to know who the current Vice-President is. What is theprobability that if you drew a random sample of 20 people from that population of 1000, at most 2 of those sampled would know the name of the Vice-President ? Give your answer (a) if the sampling were done *without* replacement, (b) if sampling were done *with* replacement, and (c) give the best approximate answer you can to the probability in (b).

(3). Box 1 contains 5 gold and 1 silver coins, and Box 2 contains 2 gold and 3 silver coins. A random experiment is carried out in two stages. First a balanced (6-sided) die is thrown; then a coin is drawn at random from Box 1 if the number of dots showing on the die is 1 or 2; otherwise (if the number of dots showing on the die is 3, 4, 5, or 6) a coin is drawn at random from Box 2.

(a) Find the probability that the coin drawn at the second stage is silver.

(b) Given that a gold coin is drawn at the second stage, find the conditional probability that the coin came from Box 1.

(4). Suppose that X is the number of dots (1 to 6) showing on the uppermost face when a single balanced die is thrown. Let Y be the winnings in a gambling game played with the die, expressed as a function of X by: $Y = (X-4)^2 - 3$. Find the probability mass function of Y, the expectation of Y, and the probability that $Y > 0$.

(5). *(Give decimal answers here !)* Suppose that events A, B, and D belong to the sample space S. Assume that with respect to the probability-rule P defined on S

(i) A and B are independent, with $P(A) = 0.3$, $P(B) = 0.4$,

(ii) B and D are disjoint, with $P(D) = 0.2$, and

(iii) $P(D|A) = 0.2$.

Then find $P(D \cap A^c)$ and $P\big(D \cup (A \cap B)\big)$.
Hint: draw a Venn diagram !