Stat 400 **Sample for In-Class Test 2** **Fall, 2003**

Instructions: Do each of the following problems; each is worth 20 points. You may use calculators, a notebook sheet of notes, and the tables in the back of your books. You need not give reduced numerical answers: numerical expressions which are readily evaluated on a calculator are good enough, unless a decimal answer is specifically requested. (But do not leave the answer in the form of an integral or abstract symbol). Justify your steps wherever you can: I will be much more generous with partial credit wherever you explain clearly what you are doing.

1. *Definitions of densities, cdf's, mean, variance* A certain continuous random variable X, with values always lying between 1 and 3, has density $f(x)$ proportional to $1/x^2$ for x between $1, 3$. Find the density, cdf, mean, and variance of X.

2. *Joint probabilities for discrete r.v.'s* Suppose that you flip a biased coin independently, with probability p of coming up heads on each toss, 4 times. Let X denote the number of times you tossed *before* your first Head, and let Y denote the number of heads on the 4 tosses. Find the probability mass function of X, and the conditional probability that $Y = 2$ given that $X = 2$. Are X, Y independent ? Explain how you know.

3. *Change of variable, simulation of continuous r.v.* If we generate a r.v. Y on the computer by simulating a $Uniform[0,1]$ r.v. U_1 and then defining $Y = 1 + (U_1)^{1/3}$, then find the probability density function (with careful attention to the interval over which its argument varies) and also find the median of Y.

4. *Quantiles for special distributions* For **each** of the distributions $\mathcal{N}(6,9)$ and $Expon(4)$, find a constant c such that the respective probabilities assigned to the interval (c, ∞) is 0.1.

5: *Simulation & Interpretation of Law of Large Numbers* (a) Explain in detail how you would simulate 1000 independent and identically distributed random variables X_k with probability density $f(x) = 2x,\ 0 \le x \le 1$, (taking as given as many *iid* $Unif[0,1]$ random variables U_j as you want). (b) For the random variables simulated in (a), what is the approximate value of the random variable $n^{-1} \sum_{k=1}^{n} X_k^3$?

6: *Approximation of probability via CLT.* A balanced die (with equal probabilities of showing 1, 2, 3, 4, 5, or 6 dots) is tossed independently 600 times. On each toss, you win \$4 if 1 or 2 dots show up, you win \$1 if 3 or 4 dots show up, and you lose \$2 if 5 or 6 dots show up. Find the approximate probability that your total gain for the 600 plays of this game lies between \$550 and \$620.

7. If the r.v. X has *Exponential(2)* density, then for the random variable $Y = \sqrt{X+1}$

(a) find the distribution function of Y, and

(b) find $E(Y^2)$.

8: Suppose that $Z_1, \ldots, Z_4$ are independent $\mathcal{N}(1,4)$ random variables, and that

$$W = Z_1 - 2\,Z_2 + 3\,Z_3 - Z_4$$

(a) Find the mean and variance of W.

(b) Find the upper quartile $(= 0.75$ quantile) for the distribution of the random variable W. *(Hint: what kind of r.v. is* W *?)*

9: Suppose that two discrete random variables X, Y have joint probability mass function given by

$$p_{X,Y}(j,k) = Pr(X = j,\, Y = k) = (1 + jk)/9 \qquad \text{for} \quad j,\, k = -1,\, 0,\, 1$$

(a) Find the marginal probability mass function $p_X(\cdot)$ of X and the marginal expectation $E(Y)$ of Y.

(b) Are X, Y independent ? Why or why not ?

(c) Find the probability mass function of $X + Y$.