Stat 400 Sample for Test 3

INSTRUCTIONS: Do each of the following three problems: each counts 35 points, and 100 points is a perfect score.

#1: A data sample $X_1, \ldots, X_n$ is known to consist of continuous random variable values with density

$$f_X(x, \vartheta) \;=\; \vartheta \cdot x^{-\vartheta - 1} \;, \qquad x > 1$$

where ϑ is an unknown parameter > 1. Find an expression in terms of the data for the method of moments estimator of ϑ.

#2 (a) Suppose that $n = 120$ independent and identically distributed random variable values X_i result in sample mean and variance values $\overline{X} = 25.34$, $S^2 = 36.0$. Give a two-sided approximate 95% confidence interval for the unknown mean μ of X_i.

(b) If there were only $n = 12$ data-points in (a), with sample mean and variance as given in (a), *and* if the *rv*'s X_i can be assumed normally distributed, then find a 95% two-sided confidence interval for μ.

#3. A gambling game (roulette) has three outcomes — Red, Black, and Green — which are supposed to have respective probabilities 9/19, 9/19, and 1/19. A state casino inspector collects data on 1900 repetitions of the game, finding 860 occurrences of Red, 910 occurrences of Black, and 130 occurrences of Green.

(a) Find a 90% two-sided confidence interval for the probability p_{red} with which the outcome Red occurs on each play.

(b) Based on these data, reasoning with a 95% confidence interval, would you say that the outcome Green for the gambling game has a probability larger than the value ($p_0 = 1/19$) which it is supposed to have ? (Assume that the casino would never allow the possibility that $p < 1/19$.)