Matlab in Math 461, part six

QR decomposition

By default, matlab computes a fancier QR decomposition than that given in Lay. If A is an $m \times n$ matrix, the command `[P S] = qr(A)` will return an $m \times m$ orthogonal matrix P and an $m \times n$ upper triangular matrix S so that $A = PS$. If A has rank n, then the first n columns of P will be an orthonormal basis for the column space of A and the last $m - n$ columns will be an orthonormal basis for its orthogonal complement, which is the null space of A^T. The last $m - n$ rows of S will be zero.

In contrast, for the usual QR decomposition given in Lay, A must have rank n, Q is the first n columns of P, and R is the first n rows of S.

So the qr matlab command is more general since it applies to any matrix and gives more information. But it is not the same as the QR decomposition given in Lay, which must be extracted from its results as indicated above. Matlab gives another form of the qr command which will do this extraction for you. If A has rank n, then the command `[Q R] = qr(A,0)` will give the QR decomposition of A, in the sense of Lay.

Least squares solutions

You can find the least squares solution to $Ax = b$ using the methods in Lay. But matlab also does this for you automatically. If A is not a square matrix then `A\b` always finds a least squares solution to $Ax = b$. However, some care must be taken if A is square and not invertible since then matlab will not determine the least squares solution. You can get around this by adding a last column of zeroes to A to fool matlab, for example `B = [A zeros(6,1)]` if A is 6×6. Now calculate `x = B\b;` to find a least squares solution for $Bx = b$. Now delete the last entry of x to get the least squares solution to $Ax = b$. (The last entry in x should be zero, but actually its value does not matter since it is mutiplied by the zero column, and hence does not affect the value of Bx.)

Singular value decomposition

The command `[U S V] = svd(A)` finds the singular value decomposition of A. So if A is $m \times n$ then U and V are unitary matrices (which means orthogonal in case A is real), $A = USV^*$, and S is a diagonal $m \times n$ matrix (that is, if A has rank r then S has an $r \times r$ diagonal matrix in its upper left corner and the rest of S is zero). The diagonal entries of S are nonnegative, in decreasing order.

Other commands

If A is a matrix then `orth(A)` gives a matrix whose columns form an orthonormal basis for the column space of A. Also `Null(A)` gives a matrix whose columns form an orthonormal basis for the Null space of A.

Problems due May 11

For all the following problems let A be a random 4×4 complex matrix with rank 2 and nonzero imaginary part. Check that its rank is in fact 2 before proceeding.

Problem 1: (10) Generate a random vector b in $\mathbb{C}^4$. Find a least squares solution $\hat{x}$ to $Ax = b$. You could use the trick given above if you wanted or any other way you prefer. Compute the error vector $b - A\hat{x}$. Check that this error vector is perpendicular to the column space of A. The error is the length $||b - A\hat{x}||$. Check that the error is minimized for the least squares solution by computing the quantity $||b - Ax||$ for several random vectors x and seeing that it is larger than than the error.

Problem 2: (5) Find a QR decomposition of A and check that $A = QR$, and that Q is unitary (recall a matrix U is unitary if $U^{-1} = U^*$). Note, I am asking for matlab's QR decomposition which differs from Lay's QR factorization. (Moreover, Lay asks that the columns of A be linearly independent to have a QR factorization.) Which columns of Q will form a basis for the column space of A?

Problem 3: (5) Find the singular value decomposition of A. Also compute `orth(A)` and `null(A)` and deduce how matlab computes them.

Problem 4: (5) Compute $B = A + A^*$ and $C = A - A^*$. Note that B is Hermitian since $B = B^*$. We say C is skew Hermitian since $C = -C^*$. Find the eigenvalues of B and C. Check that the eigenvalues of B are real as predicted by theory (the complex version of Theorem 3 page 452). Make a guess as to what a similar theorem would say about the eigenvalues of a skew symmetric matrix.