Worksheet for Sections 8.6 and 8.7

SECTION 8.6

1. Suppose that the graph of f is concave downward on the interval $[a, b]$. Does this mean that the Trapezoidal Rule yields a number that is larger than, or a number that is smaller than, the integral $\int_a^b f(x)\,dx$? Draw a picture with your answer and explanation.

2. (a) Find the exact value A of $\int_0^1 x^3\,dx$ by performing the integration.

   (b) Approximate the integral in (a) by Simpson's Rule with $n = 4$, obtaining the value B. Tell what the relationship between A and B is, and confirm it by using (8) in the section.

   (c) Tell why the Trapezoidal Rule with any value of n is (should be) larger than the exact value of the given integral in (a). (*Hint:* Draw a graph of the function x^3 on the appropriate interval, and appropriate line segments corresponding to the Trapezoidal Rule.) Then corroborate the assertion by obtaining the Trapezoidal Rule value C with $n = 50$.

3. Consider $\int_{-1}^1 x^4\,dx$. Find the exact value D of the integral and Simpson's Rule's value E, where $n = 2$. By comparing the graphs of x^2 and x^4, tell why D should indeed be less than E.

SECTION 8.7

4. Find examples of functions f, g, and h that are continuous on $[1, \infty)$ and such that

   (a) $\int_1^\infty f(x)\,dx = \infty$ (b) $\int_1^\infty g(x)\,dx = 2$ (c) $\int_1^\infty h(x)\,dx$ does not converge or equal ∞ or $-\infty$.

5. (a) Suppose f and g are continuous on $[a, \infty)$. Prove that if $\int_a^\infty f(x)\,dx$ and $\int_a^\infty g(x)\,dx$ both converge, then $\int_a^\infty (f(x) + g(x))\,dx$ also converges.

   (b) Prove that if $\int_a^\infty f(x)\,dx$ and $\int_a^\infty (f(x) + g(x))\,dx$ both converge, then $\int_a^\infty g(x)\,dx$ converges.

   (c) Give an example of f and g that are continuous on $[a, \infty)$ for which $\int_a^\infty (f(x)+g(x))\,dx$ converges but *neither* $\int_a^\infty f(x)\,dx$ nor $\int_a^\infty g(x)\,dx$ converges.