Math 213 Exam 2A Fall 2015 Name________________________________

Need: compass, straightedge (ID card will suffice)

1(18). Identify each of the following statements as True or False. If it is not always true, mark it false.

True a. Alternate interior angles are congruent if and only if lines are parallel.

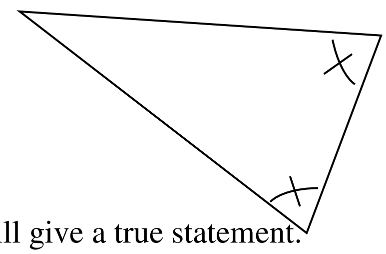
False b. A kite is a parallelogram.

True c. An equilateral triangle has three congruent angles.

True d. A figure is both a trapezoid and a kite if and only if it is a rhombus.

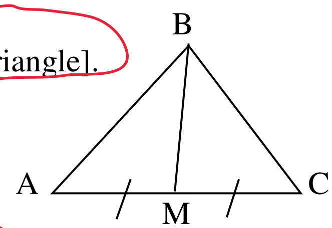
True e. A kite has at least one pair of congruent angles.

True f. The triangle at right is isosceles.



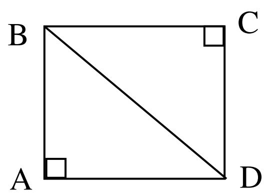
2(18). For each of the following statements, choose the condition that will give a true statement.

a. In the figure at right, $\overline{BM} \perp \overline{AC}$ if [ABC is a right triangle] [ABC is an isosceles triangle].

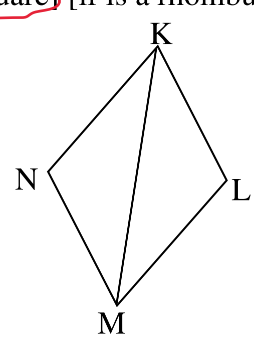


b. The base angles of a trapezoid are congruent if [it is a parallelogram] [it is isosceles].

c. In the figure below, $\overline{BD}$ bisects $< ABC$ if [ABCD is a square] [ABCD is a rectangle].



d. An equilateral quadrilateral has four congruent angles if [it is a square] [if is a rhombus].



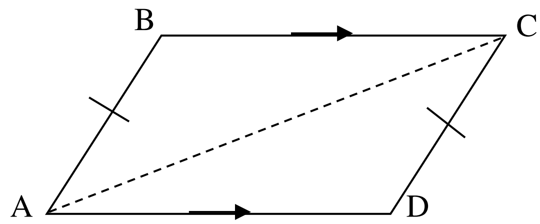
e. $\overline{KN} \parallel \overline{LM}$ if [$< LKM \cong < NMK$] [$< NKM \cong < LMK$]

f. $\overline{KN} \cong \overline{LM}$ if [KLMN is a parallelogram] [KLMN is a kite]

3(9). a. Identify the flaw in the following proof, and explain why it is incorrect.

Given: $\overline{AB} \cong \overline{CD}$; $\overline{BC} \parallel \overline{AD}$

Prove: $\overline{AB} \parallel \overline{CD}$

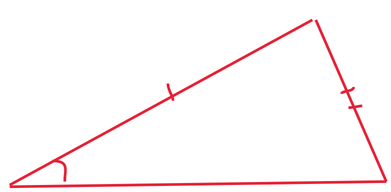


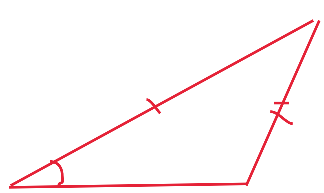
Statement	Reason
1. $\overline{AB} \cong \overline{CD}$; $\overline{BC} \parallel \overline{AD}$	1. Given
2. Draw $\overline{AC}$	2. Two points determine a line.
3. < BAC ≅ < DCA	3. Alternate interior angles are congruent
4. $\overline{AC} \cong \overline{AC}$	4. Reflexive property
5. $\Delta ABC \cong \Delta CDA$	5. SAS
6. $\overline{AD} \cong \overline{BC}$	6. CPCTC
7. ABCD is a parallelogram	7. Opposite sides of a quadrilateral are congruent if and only if it is a parallelogram
8. $\overline{AB} \parallel \overline{CD}$	8. Def'n parallelogram

<BAC and <DCA are not alternate interior angles in step 3.

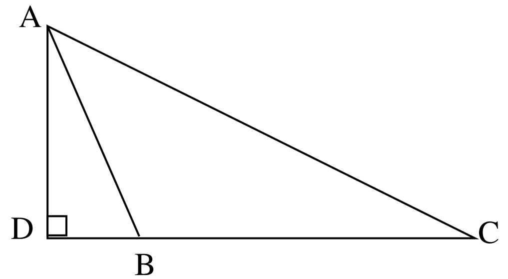
b. Can the flaw be fixed? If yes, how? If not, provide a counterexample. (Hint: Homework DR Packet)

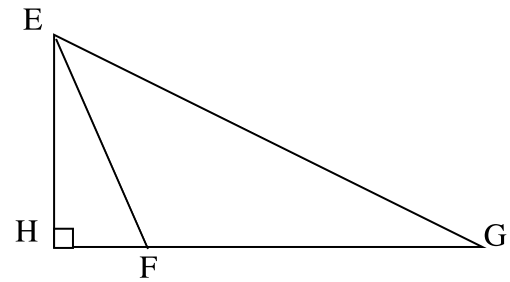
The flaw cannot be fixed, if we used alternate interior angles for that step we would be left only with SSA congruence





4(16). Fill in the missing statements and reasons in the proof below.





Given: $\Delta ABC \cong \Delta EFG$

$\overline{AD} \perp \overline{DC}$

$\overline{EH} \perp \overline{HG}$

Prove: $\overline{AD} \cong \overline{EH}$

Statement	Reason
1. $\Delta ABC \cong \Delta EFG$; $\overline{AD} \perp \overline{DC}$; $\overline{EH} \perp \overline{HG}$	1. Given
2. $\overline{AB} \cong \overline{EF}$; $< ABC \cong < EFG$	2. CPCTC
3. $< ABD + < ABC = 180°$ $< HFE + < EFG = 180°$	3. Angles forming a straight angle sum to 180
4. $\angle ABD \cong \angle HFE$	4. Transitive property and algebra
5. < ADB is right < EHF is right	5. Def'n of perpendicular lines
6. $<ADB \cong <EHF$	6. All right angles are congruent.
7. $\Delta ADB \cong \Delta EHF$	7. AAS
8. $\overline{AD} \cong \overline{EH}$	8. CPCTC

5(4). Consider the following statement: **If two triangles are congruent, then they have the same area.**

What is the *converse* of this statement?

If two triangles have the same area, then they have the same area

6. Complete the following. Use compass and straightedge (ID card) to carry out a,b,d, and f.

a. Construct the segment connecting P and Q below.

b(4). Construct the perpendicular bisector m of $\overline{PQ}$; Extend m so it intersects with the given line l. **Leave your arcs visible so your process is clear.**

c. Name the intersection of m and l point R. Name the intersection of $\overline{PQ}$ and m point S.

d. Construct segments $\overline{PR}$ and $\overline{QR}$

e(12). Complete the following proof that $\Delta RPS \cong \Delta RQS$:

$\overline{PS} \cong \overline{QS}$ because S bisects the segment $\overline{PQ}$

$\overline{PQ}$ ⊥ $\overline{RS}$ because m is the perpendicular bisector of $\overline{PQ}$ and $\overline{RS}$ is on m

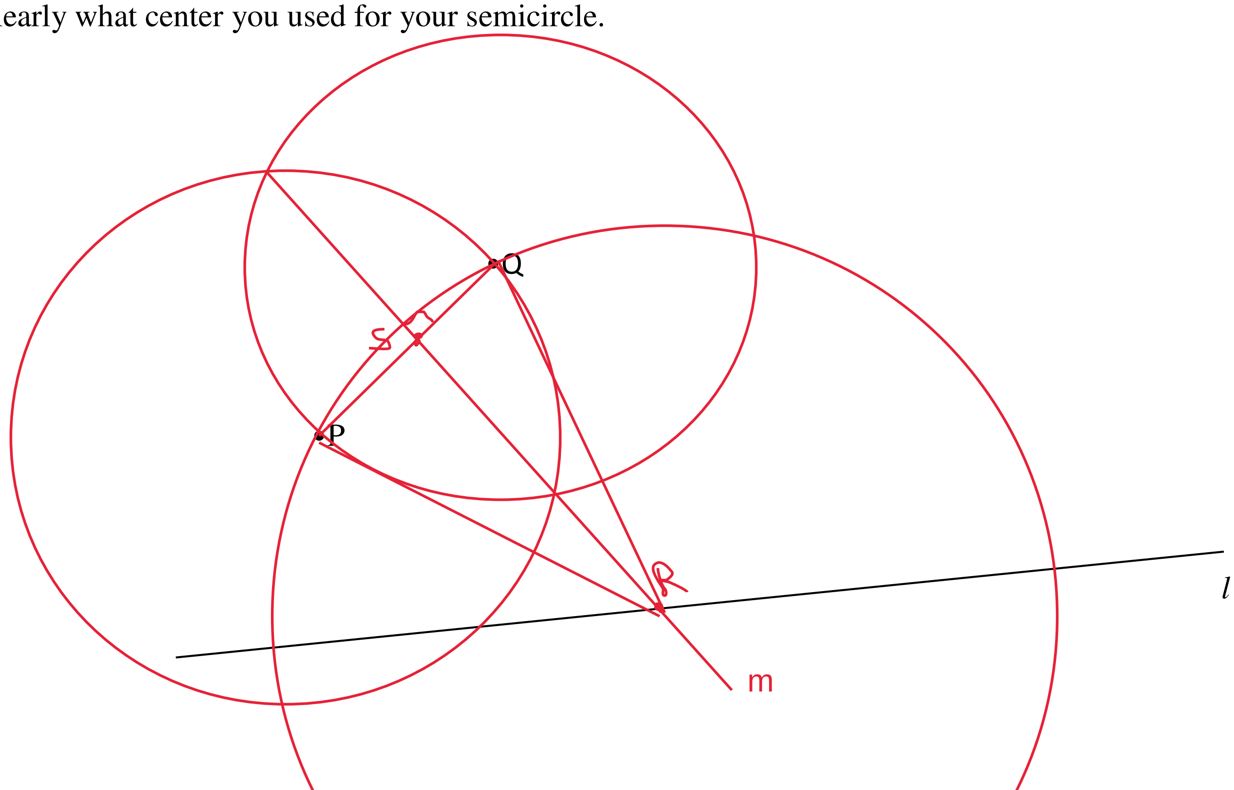
So < RSP and < RSQ are right angles by def'n of perpendicular lines

and therefore <RSP ≅ <RSQ since all right angles are congruent.

$\overline{RS} \cong \overline{RS}$ by the reflexive property.

Thus $\Delta RPS \cong \Delta RQS$ by SAS.

f(4). Construct a semicircle with its center on line l and passing through the points P and Q. Identify clearly what center you used for your semicircle.



g(3). Why did you choose the center you did? Explain based on the definition of a circle.

The center is at R since R is on l and is equidistant from Q and P by CPCTC.

7(12). Consider the following list of shapes:

A. Parallelogram
B. Square
C. Rhombus
D. Rectangle
E. Trapezoid
F. Isosceles Trapezoid
G. Kite

a. For which of the quadrilaterals listed above are the diagonals always congruent (identify by letter)?

B, D, F

b. For which of the quadrilaterals listed above do the diagonals always bisect each other?

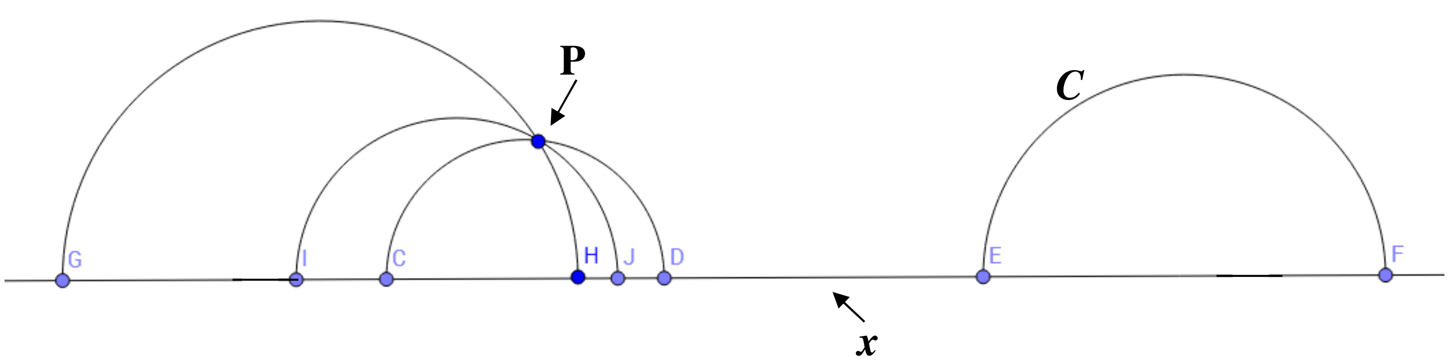
A, B, C, D

c. For which of the quadrilaterals listed above are the diagonals always perpendicular?

B, C, G

Extra Credit: In the unusual geometry pictured below, "lines" are actually semi-circular arcs with centers on the line *x*. As you can see from the figure below, given an arc *C* and a point P not on *C*, it is possible to draw many semi-circular arcs through P which do not intersect *C*.

Which one of our axioms is this fact inconsistent with?



The axiom that given a line and a point not on that line there is exactly one parallel line through that point