MATH 221 (Washington) Sample Exam 1b

1. (20 points) Evaluate the following:

(a) $\int x \cos(6x)\, dx$

(b) $\int_1^2 (2x+3)(x^2+3x)^6\, dx$

(c) $\int x^4 \ln(x)\, dx$

(d) $\int \frac{x^2}{(x^3+7)} dx$

2. (20 points) Find the derivative of each of the following functions:

(a) $x^3 \cos(x^5)$ (b) $\frac{\cos(3x)}{\sin(4x)}$ (c) $\sin(x^2+5x+3)$

3. (15 points) Consider the following right triangle.
(Imagine a triangle with hypotenuse 13 and legs 5 and 12. The angle t is where the sides of lengths 5 and 13 meet.)
Evaluate the following (express your answers as fractions):
$\sin(t) =$ $\cos(t) =$ $\tan(t) =$

4. (25 points: 15+5+5)

(a) Use the trapezoidal rule with $n = 3$ to approximate the following integral: $\int_0^6 \sqrt{x^3+1}\, dx$

(b) The graph of $y = \sqrt{x^3+1}$ is as follows:
(The graph is a curve bending upwards.)
Draw the trapezoids that are being used in the approximation in part (a).
(c) Using the graph from part (b), determine whether the actual value of the integral is greater than, equal to, or less than the approximation from part (a). Clearly state which is bigger and which is smaller. Explain why.

5. (20 points) The number of hours of daylight per day t weeks after the beginning of the year $(0 \leq t \leq 52)$ is given by

$$f(t) = 12 + 3\sin\left(\frac{\pi(t-12)}{26}\right).$$

(a) Find the value of t when the days are longest and the value of t when the days are shortest. Clearly indicate which answer is which.
(b) When $t = 38$ weeks, how fast is the number of daylight hours decreasing (your answer will be in hours per week)?
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The following formulas might be useful:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}, \qquad \int u\, dv = uv - \int v\, du, \qquad \int fg\, dx = fG - \int f'G\, dx$$

$$\text{(trapezoidal)}\ [f(a_0) + 2f(a_1) + \cdots + 2f(a_{n-1}) + f(a_n)]\frac{\Delta x}{2}$$