**MATH 221 (Washington) Exam 3 December 9, 2008 Name:** _____

**1.** (10 points) Find the value of $k$ such that $f(x) = kx^2$ is a probability density function for $1 \leq x \leq 2$?

**2.** (20 points: 10+10) Consider a random variable $X$ with density function $f(x) = \frac{1}{8}x$ for $0 \leq x \leq 4$.

(a) Compute $\Pr(1 \leq X \leq 3)$.

(b) Compute $E(X)$.

**3.** (10 points) Determine the third Taylor polynomial of $f(x) = \ln(x+1)$ at $x = 0$.

**4.** (10 points) Use one step of the Newton-Raphson algorithm to find an approximate solution to $x^3 - x - 2 = 0$. Start with $x_0 = 1$.

**5.** (15 points: 10+5) Evaluate the following sums:

(a) $5 + \frac{5}{3} + \frac{5}{9} + \frac{5}{27} + \frac{5}{81} + \cdots$

(b) $\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \cdots$

**6.** (10 points) Suppose that cars have an average life span of 10 years, and that these life spans are exponentially distributed. Find the probability that a car lasts more than 12 years.

**7.** (15 points: 10+5)

(a) The scores on a test are normally distributed with mean $\mu = 1000$ and standard deviation $\sigma = 200$. Find the probability that an individual student scores between 900 and 1200.

(b) Let $Z$ be the standard normal random variable. Find $\Pr(-\infty < Z \leq -2)$.

**8.** (10 points) Mr. Pesce is counting fish in a stream. The number he counts in a minute satisfies a Poisson distribution. The average number of fish he sees in a minute is 4. What is the probability that the number of fish he sees in a randomly chosen minute is less than or equal to 2 ?

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The following formulas might be useful:

(Newton-Raphson) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

(Taylor) $f(a) + f'(a)(x-a) + \cdots + \dfrac{f^{(n)}(a)}{n!}(x-a)^n$

(Normal) $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$, $-\infty < x < \infty$

(Exponential) $f(x) = k\,e^{-kx}$, $0 \leq x < \infty$, $\quad E(X) = \dfrac{1}{k}$

(Poisson) $p_0 = e^{-\lambda}$, $\quad p_n = (\lambda^n/n!)e^{-\lambda}$, $\quad E(X) = \lambda$