MATH 221 (Washington) Review Problems for Exam 3

1. Compute the expected value and the variance of the random variable X with the following probability table:

value of X	1	2	3
probability	0.4	0.3	0.3

2. Find the Taylor series at $x = 0$ for $f(x) = \dfrac{1}{1+x^3}$. Give all terms through the term involving x^{12}.

3. Suppose that a machine part has an average life span of 5 years and these life spans are exponentially distributed. Find the probability that a part lasts more than 3 years.

4. Let Z be the standard normal random variable. Find $\Pr(Z \geq 1.5)$.

5. The scores on a test are normally distributed with mean $\mu = 500$ and standard deviation $\sigma = 200$. Find the probability that an individual student scores between 600 and 800.

6. Let $f(x) = 3x^2$ for $0 \leq x \leq 1$.

(a) Show that $f(x)$ is a probability density function.

(b) Let X be the random variable with density function $f(x)$. Compute $E(X)$.

(c) Compute $Pr(.3 \leq X \leq .6)$.

7. Find the 3rd Taylor polynomial at $x = 1$ for $f(x) = e^x$.

8. There is 1 second between the time a bouncing ball first hits the floor and the second time it hits the floor. There are .76 seconds between the second time and the third time it hits the floor. There are $(.76)^2$ seconds between the third time and the fourth time, and each subsequent bounce takes .76 times the previous bounce. How long does the ball bounce (start timing from the first time it hits the floor)?

9. Express $0.23232323\ldots$ as the ratio of two integers.

10. Use the integral test to determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges or diverges.

11. Let $f(x) = x^2 - 6$. Start with $x_0 = 2$. Use the Newton-Raphson algorithm to find a better approximation x_1 for a solution of $f(x) = 0$.

12. The number of cars that drive through an intersection in an hour is a Poisson random variable with $\lambda = 10$. Find the probability that at least 1 car drives through the intersection in an hour.

13. Calculate (a) $\dfrac{.96}{.28} \times 28$ (b) $(.02)^3$ (c) $\dfrac{1/2}{3/7}$

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The following formulas might be useful:

(Taylor) $f(a) + f'(a)(x-a) + \cdots + \dfrac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$

$E(X) = a_1 p_1 + \cdots + a_n p_n, \quad Var(X) = (a_1 - m)^2 p_1 + \cdots + (a_n - m)^2 p_n$

$E(X) = \int_A^B x f(x)\, dx, \quad Var(X) = \int_A^B x^2 f(x)\, dx - E(X)^2$

(Normal) $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$

(Exponential) $f(x) = k\, e^{-kx}, 0 \leq x < \infty, \quad E(X) = \dfrac{1}{k}$

(Poisson) $p_0 = e^{-\lambda}$, $p_n = e^{-\lambda}\frac{\lambda^n}{n!}$, $n = 1, 2, 3, \ldots$

(Newton-Raphson) $x_{n+1} = x_n - f(x_n)/f'(x_n)$