

**Instructions:** You should have six answer sheets: number them 1 through 6, and write your name and section number on each sheet. Each sheet is for answers two or more problems (possibly unrelated). You may use both sides of the same sheet. If you need more space, ask for another sheet. Do not carry a solution from one numbered problem onto a sheet for a different numbered problem. *Show your work.*

1. (ANSWER SHEET 1) [20] Find:

a.  $\int x \cos(x^2 + 1) dx$

b.  $\int \sqrt{x} \ln x dx$

c.  $\int_0^{\infty} x e^{-x^2} dx$

2. (ANSWER SHEET 2) [15] The probability density function of a continuous random variable  $X$  is  $f(x) = 12x^2(1 - x)$  for  $0 \leq x \leq 1$ .

a. State two properties must  $f(x)$  have in order to be a probability density function. [Do not check them for  $f(x)$ .]

b. Find  $E(X)$ , and then find  $\text{Var}(X)$ . Label each answer.

3. (ANSWER SHEET 3)

a. [5] Suppose that the relief time for an analgesic drug is an exponential random variable  $X$ , with a mean of 10 minutes. Determine the probability that the relief time is greater than 20 minutes.

b. [5] The personnel office at a company hires part-time student workers. On a typical day, an average of 4 students visit the office. Assume that the number  $X$  of students that visit the office is a Poisson random variable. Estimate the probability that on a typical day *less than four* students visit the office. Use four decimal places in your calculations.

c. [5] Use the midpoint rule with  $n = 5$  to set up an estimate for value of the definite integral  $\int_3^4 \frac{x}{x+1} dx$ . Fill in all the numerical quantities, ready for the final arithmetic, but do **not** perform this arithmetic.

4. (ANSWER SHEET 4)

a. [10] Write out the Taylor series for  $e^x$ , displaying at least the first four terms. Then use this series to produce the Taylor series at  $x = 0$  for the following two functions: (Display the first three nonzero terms in each series.)

(i)  $e^{x^2}$       (ii)  $2xe^{x^2}$

b. [5] Determine whether the following series is convergent, and if it is, find its sum:

$$\frac{5^2}{3} - \frac{5^3}{3^3} + \frac{5^4}{3^5} - \frac{5^5}{3^7} + \frac{5^6}{3^9} - \dots$$

5. (ANSWER SHEET 5)

a. [5] Write the third Taylor polynomial of  $f(x) = x^{3/2}$  at  $x = 4$ .

b. [10] Find all solutions of the differential equation  $y' + 2y = 1 + e^{-t}$ . Then find the solution that satisfies  $y(0) = 4$ .

6. (ANSWER SHEET 6) (Use both sides of the answer page, if needed.)

- a. [5] Use one step ( $n = 1$ ) of Newton's method to estimate the zero of  $x^4 - 2x^3 - 3$  that is close to  $x_0 = 2$ .
- b. [5] A retired couple arranges to make an endowment to the University of Maryland. The couple makes a deposit of \$500,000 into a bank account that pays 4% annual interest, compounded continuously. The bank agrees to make continuous payments to the University at the rate of \$20,000 per year.  
Let  $f(t)$  be the amount of money in the account at time  $t$  (in years). Write a differential equation that is satisfied by  $f(t)$ , and list the initial condition. (Do not solve the equation; do not sketch solutions.)
- c. [10] Without solving the equation, use the qualitative theory of differential equations to sketch solutions of  $y' = y^2 - 4y - 5$  that satisfy the initial conditions  $y(0) = 0$ ,  $y(0) = 4$ , and  $y(0) = 6$ . (Include the constant solutions.) Include both a  $yz$ -graph and a  $ty$ -graph. Identify asymptotes and inflection points (if any).