Math 221 **Final Exam** **May 13 , 2006**

Instructions: You have 10 answer sheets. Write your name and section number on each sheet. Number the sheets from 1 to 10 and work the problems for Answer Sheet 1 on Answer Sheet 1, the problems for Answer sheet 2 on Answer Sheet 2, etc. Use the back of a sheet if necessary. *Basic formulas can be found on the back of this page.* **You must show work to receive credit.**

ANSWER SHEET 1 (5 POINTS)

Find an equation for the tangent line of the graph of the function $f(x) = (x+1)\cos(x^2+2x)$ at the point where $x = 0$.

ANSWER SHEET 2 (8 POINTS)

Use the midpoint rule M, the trapezoidal rule T, and Simpson's rule S with two intervals to approximate the integral

$$\int_{-\pi/2}^{\pi/2} \cos^2(x)dx$$

ANSWER SHEET 3 (10 POINTS)

Compute the integrals

$$(a) \int x\cos(5x+1)dx, \quad \text{and} \quad (b) \int_0^\infty \frac{3}{(4x+5)^2}dx$$

ANSWER SHEET 4 (12 POINTS)

Solve the differential equation $y' = y^2 - e^{3t}y^2$, with initial condition $y(0) = 1$.

ANSWER SHEET 5 (15 POINTS)

(a) Solve the differential equation $y' + \frac{1}{4}y = 4$, with initial condition $y(0) = 0$ and also with the initial condition $y(0) = 16$.

(b) A patient receives a continuous infusion of a drug into the bloodstream, at the rate of 4 mg per day. The patients body eliminates the drug at the daily rate of 25 % of the drug present in the system. Let $y = f(t)$ represent the amount of drug present in the body at time t (with time measured in days). Set up the differential equation satisfied by y.

(c) Determine approximately how many mg of the drug is in the bloodstream after a long time.

ANSWER SHEET 6 (15 POINTS)

(a) Find the sum of the infinite series $\sum_{n=0}^{\infty} 4(\frac{3}{4})^n$

(b) A patient receives 4 mg of a certain drug, once a day, at the same time each day. In one full day, the body eliminates 25 % of the amount of drug present in the system.

(i) Write an expression that gives the amount of drug in the patient's body immediately after the third dose has been given (two days after the initial dose)

(ii) Estimate the approximate total amount of drug present in the patients body after many weeks of treatment, immediately after a dose is given.

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Answer Sheet 7 (8 points)

(a) Compute the third Taylor polynomial for the function $f(x) = \ln x$ at $x = 1$.
(b) Find the coefficient to $(x-1)^{100}$ in the 100'th Taylor polynomial for $f(x) = \ln x$ at $x = 1$.

Answer Sheet 8 (10 points)

(a) Find the values of k and C that makes $F(x) = k\sqrt{x} + C$ the cumulative distribution function of a random variable X with values between 1 and 4.
(b) Find the expected value of X.

Answer Sheet 9 (10 points)

Suppose the life span of the automatic transmission of a certain type of automobile is normally distributed with expected value $\mu = 150{,}000$ miles and standard deviation $\sigma = 10{,}000$ miles. What is the probability that a transmission will last for at least 165,000 miles? (*See attached Table*).

Answer Sheet 10 (7 points)

Assume that the number X of typographical errors per page of a certain newspaper is a Poisson random variable and the probability is 0.5 that there are no errors per page.
(a) What is the probability that a page has more than 1 error?
(b) What is the average number of errors per page?

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Useful Formulas

Midpoint Rule: $M = (f(x_1) + f(x_2) + \ldots + f(x_n))\Delta x \approx \int_a^b f(x)dx$

Tapezoidal Rule: $T = (f(a_0) + 2f(a_1) + \ldots + 2f(a_{n-1}) + f(a_n))\frac{\Delta x}{2} \approx \int_a^b f(x)dx$

Simpson's Rule: $S = \frac{2}{3}M + \frac{1}{3}T \approx \int_a^b f(x)dx$

Euler: $y_{n+1} = y_n + g(t_n, y_n)h$

Newton - Raphson: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Expected value and Variance: $E(X) = \int xf(x)dx$, $Var(X) = \int x^2 f(x)dx - E(X)^2$

Normal distribution: $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})}$, $-\infty < x < \infty$, $E(X) = \mu$, $Var(X) = \sigma^2$

Exponetial distribution: $f(x) = ke^{-kx}$, $E(X) = \frac{1}{k}$, $Var(X) = \frac{1}{k^2}$

Poisson distribution: $p_0 = e^{-\lambda}, p_n = \frac{\lambda^n}{n!}e^{-\lambda}, n = 1, 2, \ldots,$ $E(X) = \lambda$, $Var(X) = \lambda$

Geometric random variable: $p_n = p^n(1-p), n = 0, 1, 2, \ldots,$ $E(X) = \frac{p}{1-p}, Var(X) = \frac{p}{(1-p)^2}$

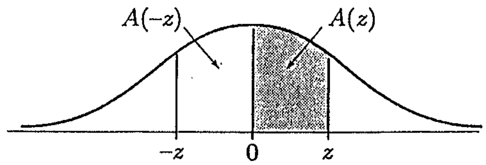

TABLE 1 Areas Under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3820
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998