MATH 221 (Washington) Sample Exam 2

1. The rate of spread of a piece of news is proportional to the product of the number of people who have heard the news and the number of people who have not heard the news. Suppose there are 500,000 in the population. Write a differential equation that is satisfied by the number of people who have heard the news.
2. The rate at which a drug leaves the bloodstream is proportional to the amount of the drug in the bloodstream. Write a differential equation satisfied by the amount of the drug in the bloodstream at time t.
3. Solve the differential equation $y' = -6y + te^{-t}, \quad y(0) = 3.$
4. You have a bank account that earns 6% interest compounded continuously. You continuously withdraw money from the account at the rate of \$3000 per year. Write a differential equation that is satisfied by the amount of money in the account at time t.
5. Solve the differential equation $y' = (3t^2 + 12t)/y, \quad y(0) = 6.$
6. Consider the differential equation $y' = .02(y^2 - 8y)$. Graph the solutions with $y(0) = 6$ and with $y(0) = 10$. Do not graph any other non-constant solutions.
7. Consider the following weird investment plan. If y is the amount of money in the plan, then y satisfies the differential equation $y' = -y(y - 300)(y - 500)$. Suppose person A invests 300.01 and person B invests 299.99. After 100 years, what is the approximate difference between what A has in the account and what B has in the account?
8. Calculate (a) $(1/2)/(7/8)$ (b) $.5 \times .24$ (c) $.003 + .0124$
****

The following formulas might be useful:

$$\int fg\,dx = fG - \int f'G\,dx, \qquad \int u\,dv = uv - \int v\,du$$

$$A(t) = \int a(t)\,dt, \qquad y = e^{-A(t)}\left[\int e^{A(t)}b(t)\,dt + C\right]$$