1. (14 points)
Let $f$ be the function $f(x) = x^2 - 8 \ln x$ with domain $[1, 10]$.
(a) (4 pts) What properties of $f$ and its domain guarantee that $f$ will assume maximum and minimum values?
(b) (10 pts) What are the maximum and minimum values assumed by $f$ on its domain?

2. (10 points)
Find the equation of the tangent line to the curve $4e^{2x} - y^2 = 0$ at the point $(0, 2)$.

3. (15 points)
Let $f$ be the function with domain $[0, 2]$ defined by $f(x) = \sqrt{2x + 1}$.
(a) (7 pts) Compute the left endpoint Riemann sum estimate $\sum_{i=1}^{4} f(x_{i-1}) \Delta x$ of $\int_{x=0}^{2} f(x) \, dx$ when $n = 4$. (Do not simplify the expression you obtain from the definition.)
(b) (5 pts) Draw the graph of $f$ and the rectangles corresponding to this Riemann sum.
(c) (3 pts) Is this Riemann sum greater or smaller than $\int_{x=0}^{2} f(x) \, dx$?

4. (14 points)
Let $f$ be the function on $[0, 4]$ defined by $f(x) = (2x + 1)^{1/4}$. Let $R$ be the “region under the curve”, i.e. the set of points $(x, y)$ such that $0 \leq x \leq 4$ and $0 \leq y \leq f(x)$. Let $S$ be the solid of revolution obtained by rotating $R$ about the $x$-axis.
What is the volume of $S$?
**** THERE ARE MORE PROBLEMS ON THE OTHER SIDE. ****
5. (18 points)
   (a) (8 pts) Compute the average value of the function $f(x) = \sec^2(x)$ over the interval $[0, \pi/4]$.
   (b) (10 pts) Evaluate the definite integral
   \[
   \int_{x=\pi/4}^{\pi/2} \sqrt{\sin x \cos x} \, dx
   \]

6. (14 points)
   Let $s(t)$ be the position of a certain object at time $t$. Suppose its velocity at time $t$ is $e^{2t}$, and suppose $s(0) = 1$.
   What is the position of the object at time $t = 3$?

7. (15 points) According to Poiseuille’s laws, the velocity $v$ of blood in a blood vessel is given by $v(r) = k(R^2 - r^2)$, where $R$ is the (constant) radius of the blood vessel, $r$ is the distance of the flowing blood from the center of the blood vessel, and $k$ is a positive constant.
   
   Given $R$, let $Q(R)$ be the total blood flow (in milliliter per minute) in the vessel. For $n$ a positive integer, $Q(R)$ is approximated by a sum
   \[
   \sum_{i=1}^{n} v(r_i)2\pi r_i \Delta r
   \]
   in which $\Delta R = R/n$ and $r_i = i\Delta r$. As $n$ goes to $\infty$, the sum converges to $Q(R)$.
   (a) (5 pts) Write a definite integral which equals $Q(R)$.
   (b) (10 pts) Compute the definite integral.