1. **(10 points)** For the initial value problem \( \frac{dy}{dx} = -3xy + 2 \), \( y(0) = 1 \) use Euler’s method with step size 0.1 to estimate \( y(0.2) \).

2. **(13 points)**
   (a) (10 pts) Solve the initial value problem \( \frac{dy}{dt} = y^2 \), \( y(1) = 1 \).
   (b) (3 pts) What is the largest \( b \) (either a number or \( \infty \)) such that the solution is valid on the interval \([1, b)\)?

3. **(12 points)** Answer True or False. No comment required.
   (a) If \( A \) and \( B \) are \( 2 \times 3 \) matrices, then \( A + B \) is well defined.
   (b) If \( A \) and \( B \) are \( 2 \times 3 \) matrices, then \( AB \) is well defined.
   (c) If \( A \) and \( B \) are \( 2 \times 2 \) matrices, then \( AB = BA \).
   (d) If \( A, B, C \) are matrices such that \( AB = AC \), then \( B = C \).

4. **(12 points)** Suppose the following matrix is the augmented matrix of a system of linear equations in \( m \) equations in \( n \) variables:

\[
A = \begin{pmatrix}
0 & -2 & 3 & 2 & 1 & 1 \\
0 & 0 & 0 & 2 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}.
\]

   (a) (4 pts) What is \( m \)? What is \( n \)?
   (b) (3 pts) How many solutions does the system have?
   (c) (2 pts) How many free variables are there?
   (d) (3 pts) Give an example of an augmented matrix for a system of linear equations with no solution.

5. **(12 points)** Suppose \( y = (y_1, y_2) \), \( x = (x_1, x_2) \) and \( y = f(x) \) is defined by

\[
y_1 = (x_1)^2 x_2 \quad \text{and} \quad y_2 = 3x_1 x_2.
\]

   (a) (7 pts) Compute the matrix which is the derivative of \( y \) with respect to \( x \) at the input \((x_1, x_2) = (1, -1)\).
   (b) (5 pts) Use the derivative to approximate \( f(1.1, -0.8) - f(1, -1) \).
6. (13 points) At time \( t = 0 \), a tank holds 100 gallons of water that contain 20 pounds of salt. A salt solution (2 pounds of salt per gallon) flows into the tank at the rate of 8 gallons per hour, and the solution in the tank flows out at the same rate. The amount \( x \) of salt (in pounds) at time \( t \) (in hours) is assumed to satisfy a differential equation of the form \( \frac{dx}{dt} = kx + b \), where \( k \) and \( b \) are constants.
(a) (4 pts) What are \( k \) and \( b \)?
(b) (7 pts) Find a formula for \( x \) as a function of \( t \).
(c) (2 pts) As \( t \to \infty \), what value does \( x(t) \) approach?

7. (13 points) The system of differential equations
\[
\begin{align*}
\frac{dx_1}{dt} &= 3x_1 - 2x_1x_2 \\
\frac{dx_2}{dt} &= -4x_2 + 5x_1x_2
\end{align*}
\]
is chosen such that \( x_1(t) \) and \( x_2(t) \) model the sizes of two populations as a function of time.
(a) (2 pts) What is the equilibrium point of the system at which \( x_1 \neq 0 \) and \( x_2 \neq 0 \)?
(b) (8 pts) Suppose at \( t = 0 \) that \( x_1 = 1 = x_2 \). Find an equation (expressed in terms of \( x_1 \) and \( x_2 \), without using derivatives or \( t \)) satisfied by the solution \((x_1(t), x_2(t))\) for all \( t \).
(c) (3 pts) For the initial condition \( x_1(0) = 0.1 \) and \( x_2(0) = 0.1 \), what is the long term behavior of \( x(t) \) as \( t \) increases?

8. (19 points) For the system of differential equations
\[
\begin{align*}
x_1 + 4x_2 &= \frac{dx_1}{dt} \\
3x_1 + 2x_2 &= \frac{dx_2}{dt}
\end{align*}
\]
do the following.
(a) (2 pts) Presented as a matrix equation, this system takes the form \( Mx = \frac{dx}{dt} \), where \( x = (x_1, x_2) \) and \( \frac{dx}{dt} = \left( \frac{dx_1}{dt}, \frac{dx_2}{dt} \right) \). What is the matrix \( M \)?
(b) (8 pts) Find a matrix \( P \) and a diagonal matrix \( D \) such that \( P^{-1}MP = D \).
(c) (3 pts) Find the general solution \( x(t) \) to the given linear system. (It should include two undetermined constants, \( C_1 \) and \( C_2 \).)
(d) (3 pts) Compute \( P^{-1} \).
(e) (3 pts) Assuming the initial condition \( x_1 = 1 \) and \( x_2 = 2 \) at \( t = 0 \), compute the constants \( C_1 \) and \( C_2 \) in your general solution.