I will disregard the hint and understand the system in a straightforward way without an a priori guess at good coordinates.

For $i = 1, 2$, let $x_i$ be the distance down from the support of the first mass (particle). Let $F_i$ denote the force acting on the $i$th particle. We then have

$$F_1 = m_1 g - k_1 (x_1 - L_1) + k_2 (x_2 - x_1 - L_2)$$
$$F_2 = m_2 g - k_2 (x_2 - x_1 - L_2) .$$

Note that the forces acting on a particle comes only from direct action: a spring connected to the particle, and gravity.

Next we find the values of $x_1$ and $x_2$ at equilibrium by setting the forces equal to zero and solving. From the equation for $F_2$, we find at equilibrium that

$$\frac{m_2 g}{k_2} = x_2 - x_1 - L_2$$

Substituting this expression for $x_2 - x_1 - L_2$ into the equation $0 = F_1$, we get

$$0 = m_1 g - k_1 (x_1 - L_1) + k_2 \left( \frac{m_2 g}{k_2} \right), \text{ so}$$
$$k_1 (x_1 - L_1) = m_1 g + m_2 g .$$

Next we solve the last equation and our earlier equation to get the values $x_i = b_i$ at equilibrium:

$$b_1 = L_1 + \frac{(m_1 + m_2)g}{k_1}$$
$$b_2 = L_1 + \frac{(m_1 + m_2)g}{k_1} + L_2 + \frac{m_2 g}{k_2} .$$

Set $y_i = x_i - b_i$ (so, $y_1 = 0 = y_2$ at equilibrium). Now for $i = 1, 2$, substitute $y_i + b_i$ for $x_i$ in the equation for $F_i$, and simplify. This shows that, indeed,

$$F_1 = -k_1 y_1 + k_2 (y_2 - y_1)$$
$$F_2 = -k_2 y_2 .$$

Typeset by A4M-TeX