AMSC/CMSC 666 Dr. Wolfe ASSIGNMENT #6 Due May 11, 2009

1. Exercise 5, p.720. **Stoer and Bulirsch**.
2. Exercise 6, p.720. **Stoer and Bulirsch**.
3. Exercise 8, parts (a), (b), (c) p.721. **Stoer and Bulirsch**.
4. (MATLAB) Let A be the matrix defined in Ex. 11, p.722, **Stoer and Bulirsch**. Let $\mathbf{b} = [2, 1, 2, 2, 1, 2]^T$. The system $A\mathbf{x} = \mathbf{b}$ has the solution $\mathbf{x} = [1, 1, 1, 1, 1, 1]^T$. Solve the system using Jacobi, Gauss-Seidel and overrelaxation for several values of the relaxation parameter ω. In each case find the spectral radius of the iteration matrix. Try to find the optimal ω by trial and error.
5. (MATLAB) Solve the system of problem 4 by the method of Conjugate Gradients.
6. Let A and B be $n \times n$ real matrices with A nonsingular. Consider solving the system

$$Az_1 + Bz_2 = b_1, \quad Bz_1 + Az_2 = b_2$$

with $z_1, z_2, b_1, b_2 \in \mathbf{R}^n$.

(a) Find necessary and sufficient conditions for the convergence of the iteration method

$$Az_1^{(m+1)} = b_1 - Bz_2^{(m)}, \quad Az_2^{(m+1)} = b_2 - Bz_1^{(m)}, \quad m \geq 0.$$

(b) Repeat part (a) for the iteration method

$$Az_1^{(m+1)} = b_1 - Bz_2^{(m)}, \quad Az_2^{(m+1)} = b_2 - Bz_1^{(m+1)}, \quad m \geq 0.$$

Compare the convergence rates of the two methods.