SOLUTIONS: PROBLEM SET 1 FROM SECTION 1.2

3. (I did this problem by mistake, but left it in as another example) Notice that what is to be proved inductively is not an equation but an inequality. Let S be the set of positive numbers for which the inequality is true.

For $n = 1$, the left hand side and the right hand side of the inequality are both equal to one. Since the stated inequality is not strict, it is true in this case, so $1 \in S$.

Suppose $n \in S$. Then

$$\sum_{k=1}^{n+1} \frac{1}{k^2} = \sum_{k=1}^{n} \frac{1}{k^2} + \frac{1}{(n+1)^2} \leq 2 - \frac{1}{n} + \frac{1}{(n+1)^2} = 2 - \frac{1}{n+1} - \frac{1}{n(n+1)^2} \leq 2 - \frac{1}{n+1}.$$

It now follows that $n + 1 \in S$, completing the proof by induction.

4. The first few values of the sum are $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, leading to the conjecture that

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}.$$

Let S be the set of integers for which this is true. We have already observed that $1 \in S$ before formulating the conjecture.

Now let us assume that $n \in S$. Then

$$\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n+1}{n+2},$$

showing that $n + 1 \in S$.

12. Once again, let S be the set of integers for which the equation is true. If we set $n = 1$, both sides of the equation reduce to 1; hence $1 \in S$.

Now assume $n \in S$. Then we have

$$\sum_{j=1}^{n+1} j \cdot j! = (n+1)! - 1 + (n+1)(n+1)! = (n+1)!(n+2) - 1 = (n+2)! - 1,$$

so that $n + 1 \in S$, completing the inductive proof.