SOLUTIONS: PROBLEM SET 12 FROM SECTION 4.5

2.

(a) The determinant of the coefficients is 7. It follows that the two equations are either inconsistent or equivalent $\pmod 7$. In this case they are equivalent, so the solutions are given by $x \equiv 6-5y \pmod 7$ with y arbitrary. Since there are 7 incongruent values for y, there are seven incongruent solutions for this system of congruences.

(b) Again the determinant is 7, but this time the two congruences are inconsistent, so there is no solution.

6. The square of this matrix is

$$\begin{pmatrix} 27 & 286 \\ 26 & 495 \end{pmatrix}$$

Reducing the entries $\pmod{26}$, we obtain the identity matrix. 8.

(a) The matrix is involutory, and so is its own inverse.

(b)

$$\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}$$