SOLUTIONS: PROBLEM SET 13 FROM SECTION 46.1

10. $(11, 6) = 1$; hence by the little Fermat theorem, $6^{10} \equiv 1 \pmod{11}$. Since $10|2000$, $6^{2000} \equiv 1 \pmod{11}$, so the desired remainder is 1.

12. Since $2|10$, $16 = 2^4|10^6$. Consequently $2^{10^6} \equiv 1 \pmod{17}$ and the desired least positive residue is 1, as in problem 10.

20. $(a, 42) = 1$ if and only if a is relatively prime to 2,3 and 7. If a is relatively prime to 7, then $a^6 \equiv 1 \pmod 7$. If a is relatively prime to 3, then $a \equiv \pm 1 \pmod 3$ and $a^6 \equiv 1 \pmod 3$. Finally, if a is odd, then $8|\, a^6 - 1 = (a-1)(a+1)(a^4 + a^2 + 1)$. Consequently, if $(a, 42) = 1$, $a^6 - 1$ is divisible by 7, 3, and 8, and hence by 168.

28. If p and q are distinct primes, then $p^{q-1} \equiv 1 \pmod q$ and $q^{p-1} \equiv 0 \pmod q$. Hence $p^{q-1} + q^{p-1} \equiv 1 \pmod q$. Similarly $p^{q-1} + q^{p-1} \equiv 1 \pmod p$, so by the Chinese remainder theorem, $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$.