SOLUTIONS: PROBLEM SET 15 FROM SECTION 7.1

2.

(a) $\phi(100) = 40$
(b) $\phi(256) = 128$
(c) $\phi(1001) = \phi(7)\phi(11)\phi(13) = 720$
(d) 5760
(e) 829440
(f) 416084687585280000

4.

(a) 1,2
(b) 3,4,6
(c) No such integer
(d) 5,8,10,12

8. Suppose $\phi(n) = 14$. Since 15 is not prime, n cannot be a prime or twice an odd prime. Since 4 does not divide $\phi(n)$, n cannot have two distinct prime power factors for which ϕ takes an even value. It follows that n must have the form p^k or $2p^k$ for p an odd prime and $k \geq 2$. But then $p^{k-1}|\phi(n)$ from which it follows that $p = 7$ and $n = 2$. This leaves the possibilities 49 and 98, and ϕ does not take the value 14 for either of those.

16. Since ϕ is multiplicative, and $2|\phi(p^j)$ for any odd prime p and positive integer j, it follows that, if n has k distinct odd prime powers in its prime power decomposition $2^k|\phi(n)$.