SOLUTIONS: PROBLEM SET 17 FROM SECTION 7.3

2. $2^{18}(2^{19}-1)$ and $2^{30}(2^{31}-1)$.

4.
   (a) 7
   (b) $2^{17}-1$
   (c) 2047

8. "Divisor" in this problem should be interpreted as "proper divisor," where a proper divisor of n is a divisor other than n. This is a consequence of the more general fact that if d is a proper divisor of n, than $\frac{\sigma(d)}{d} < \frac{\sigma(n)}{n}$. This, in turn, follows from the facts that

- $\frac{\sigma(n)}{n}$ is multiplicative
- for all $n > 1$, $\frac{\sigma(n)}{n} > 1$,
- for all primes p, $\frac{\sigma(p^k)}{p^k}$ is an increasing function of k.

If d is a proper divisor of n, then either n has at least one prime power factor that d does not have, or at least one prime divides n to a higher power than it divides d. In either case, the result follows.

26. If $n = 2^q$, then $\sigma(n) = 2^{q+1} - 1$. If, in addition, $\sigma(n)$ is prime, then $\sigma(\sigma(n)) = \sigma(n) + 1 = 2^{q+1} = 2n$.