SOLUTIONS: PROBLEM SET 18 FROM SECTION 7.4

2.

(a) $\mu(33) = 1$
(b) $\mu(105) = -1$
(c) $\mu(110) = -1$
(d) $\mu(740) = 0$
(e) $\mu(999) = 0$
(f) $\mu(3 \cdot 7 \cdot 13 \cdot 19 \cdot 23) = -1$
(g) $\mu(\frac{10!}{5!^2}) = 0$

6. We are looking for all composite integers between 100 and 200 that are products of an odd number of distinct primes. Since the product of the five smallest primes is greater than 200, it follows that we are looking for products of three primes. We can represent such a product as $p_1p_2p_3$ with $p_1 < p_2 < p_3$. Moreover, since $2 \times 11 \times 13 = 286 > 200$, we know that $p_2 \leq 7$. Moreover, if $p_2 = 7$, $p_1 = 2$ since $3 \times 7 \times 11 = 231 > 200$. Thus the possible values for p_1p_2 are 14, 15,10 and 6, leading to the listing

$$\begin{aligned} 14p_3 &= 154, 182 \\ 15p_3 &= 165, 195 \\ 10p_3 &= 110, 130, 170, 190 \\ 6p_3 &= 114, 138, 174, 186 \end{aligned}$$

16. Since $\sum_{d|n} \phi(\frac{n}{d})$ differs from $\sum_{d|n} \phi(d)$ only in the order of summation, it follows that n is the summatory function of $\phi(n)$. Since n is obviously a multiplicative function of n, it follows that ϕ is multiplicative. Applying the Möbius inversion formula to n as the summatory function of ϕ, it follows that

$$\phi(p^t) = \sum_{j=0}^{t} p^j \mu(p^{t-j}).$$

Because $\mu(t^k) = 0$ for $k > 1$, the only two terms in this sum that make a non-vanishing contribution are the last two, giving $p^{t-1}\mu(p) + p^t\mu(1) = -p^{t-1} + p^t$.