SOLUTIONS: PROBLEM SET 19 FROM SECTION 9.1

2.

(a) $\text{ord}_{11}3 = 5$
(b) $\text{ord}_{17}2 = 8$
(c) $\text{ord}_{21}10 = 6$
(d) $\text{ord}_{25}9 = 10$

4.

(a) 3 is a primitive root $\pmod 4$.
(b) 3 is a primitive root $\pmod 5$.
(c) 3 is a primitive root $\pmod{10}$.
(d) 2 is a primitive root $\pmod{13}$.
(e) 3 is a primitive root $\pmod{14}$.
(f) 5 is a primitive root $\pmod{18}$.

6. $20 = 5 \times 4$. 4 and 5 are relatively prime, but $\phi(4) = 2$ and $\phi(5) = 4$ are not. Hence there is no primitive root $\pmod{20}$.

12. If this were true in general, it would be true for $d = \phi(n)$ and hence there would always be a primitive root, which we know to be false.

Even if we require that d be a proper divisor of $\phi(n)$, the statement is false. Consider the case $n = 65 = 5 \times 13$. $\phi(65) = \phi(5)\phi(13) = 4 \times 12 = 48$. However $[\phi(5), \phi(13)] = [4, 12] = 12$. It follows that for $(a, 65) = 1$, $\text{ord}_{65}a|12$. In particular, there is no integer a with $\text{ord}_{65}a = 24$, although 24 is a proper divisor of 48.