

SOLUTIONS: PROBLEM SET 2 FROM SECTION 1.4

10. If a and b are positive and $a|b$, then $b = ac$ for some integer c and, since a and b are positive, c is positive as well. Hence $c \geq 1$ and therefore $b = ac \geq a \cdot 1 = a$.

14. Suppose a and b are odd and positive and $b|a$. Then by the division algorithm, $a = qb + r$ with $0 < r < b$. If r is odd, we can set $s = q$ and $t = r$. If r is even, we have $a = (q + 1)b + (r - b)$. Since $0 < r < b$, it follows that $-b < r - b < 0$, and hence $0 < |r - b| < b$. Moreover, since r is even and b is odd, it follows that $r - b$ is odd. In this case, we can set $s = q + 1$ and $t = r - b$.