SOLUTIONS: PROBLEM SET 21 FROM SECTION 9.3

4.

(a) 2 is a primitive root $\pmod{121}$
(b) 2 is a primitive root $\pmod{169}$
(c) 3 is a primitive root $\pmod{289}$
(d) 2 is a primitive root $\pmod{361}$

10. The primitive roots $\pmod 5$ are 2 and 3. $\phi(\phi(25)) = \phi(20) = 8$, so there are 8 primitive roots $\pmod{25}$, four congruent to $2 \pmod 5$ and four congruent to $3 \pmod 5$. The four congruent to $2 \pmod 5$ are 2,12,17 and 22; the four congruent to $3 \pmod 5$ are 3,8,13 and 23. 7 and 18 are primitive roots $\pmod 5$ but not $\pmod{25}$.

16. Such a root exists for every odd prime p. In particular, for $p = 3$, 8 is such a primitive root. Most of you assumed the additional requirement that $r < p$. In that case, as many of you established, no such root exists for $p \leq 17$. I gave full credit for this observation.

FLASH: 14 is a primitive root $\pmod{29}$ but not $\pmod{29^2}$. This bulletin is courtesy of my colleague Larry Washington.