SOLUTIONS: PROBLEM SET 33 FROM SECTION 13.3

2.

(a) $19 \equiv 3 \pmod 4$ and so is not the sum of two squares.
(b) 25, like any perfect square is the sum of two squares, one of which is 0.
(c) $29 = 5^2 + 2^2$
(d) $45 = 6^2 + 3^2$
(e) $65 = 8^2 + 1^2$
(f) $80 = 8^2 + 4^2$
(g) 99 is not the sum of two squares because it is divisible by 11, but not by 11^2, and 11 is a prime congruent to 3 $\pmod 4$.
(h) $999 = 3^3 \times 37$ and is not the sum of two squares.
(i) 1000, like any power of 10, is the sum of two squares because $10 = 3^2 + 1^2$ is the sum of two squares.

4. Any odd number is the difference of two consecutive squares. Any number divisible by 8 is the difference of two consecutive odd squares, and any number congruent to 4 $\pmod 8$ is the difference of two consecutive even squares. On the other hand, since all squares are congruent to 1 or 0 $\pmod 4$, differences of squares are congruent only to ± 1 or to 0 $\pmod 4$.

6. All squares are congruent to 1, 4 or 0 $\pmod 8$. 7 is not congruent to the sum of any three of these $\pmod 8$, even allowing repetitions.