SOLUTIONS: PROBLEM SET 8 FROM SECTIONS 3.6 AND 4.1

3.6.2

(a) $x = 1 + 4u; \quad y = 1 - 3u$

(b) There is no solution because $(12, 18) = 6$ and 6 does not divide 50.

(c) $x = -121 - 47u; \quad y = 77 + 30u$

(d) $x = 776 + 19u; \quad y = -194 - 5u.$

(e) $x = 422 + 1001u; \quad y = -43 - 102u$

3.6.6 The relevant Diophantine equation is $18x + 33y = 549$, which reduces, on divison by 3 to $6x + 11y = 183$. We have $6 \times 2 - 11 \times 1 = 1$, which gives us the general solution of our original equation, $x = 366 - 11u; \quad y = -183 + 6u$. Setting $u = 31$, we get $x = 25; \quad y = 3$. Making this our new base solution, we obtain $x = 25 - 11t; \quad y = 3 + 6t; x + y = 28 - 5t$, so we want to make t as small as possible, consistent with making both x and y non-negative. Clearly the best we can do is $t = 2$, giving $x = 3; \quad y = 15; \quad x + y = 18$.

3.6.8 We are looking here at Diophantine equations for which the left hand side is $11x + 8y$. We have $11 \times 3 - 8 \times 4 = 1$. This enables us to write down the general solution in each case very quickly. We need only determine which solutions are non-negative.

(a) The general solution is $x = 2331 - 8u; \quad y = -3108 + 11u$ We obtain non-negative solutions for $283 \leq u \leq 291$. The smallest value of x is given by the solution $(3, 93)$; the others are $(11, 82)$, $(19, 71)$, $(27, 60,(35, 49)$, $(43, 38)$, $(51, 27)$, $(59, 16)$, and $(67, 5)$.

(b) The general solution is $(288 - 8u, -384 + 11u)$ the non-negative solutions are $(0, 12)$ and $(8, 1)$.

(c) The general solution is $(207 - 8u, -176 + 11u)$; there are no non-negative solutions in this case, so there must be an error in the bill.

3.8.14 The relevant system of equations is

$$5x + 10y + 25z = 200$$
$$x + y + z = 24.$$

We immediately divide the first equation by 5 to obtain the new system

$$x + 2y + 5z = 40$$
$$x + y + z = 24.$$

Subtracting the second equation from the first, we obtain the equation $y + 4z = 16$. We need non-negative solutions with $y + z \leq 24$. This gives, for x, y and z, the possibilities $(20, 0, 4)$, $(17, 4, 3)$, $(14, 8, 2)$, $(11, 12, 1)$ and $(8, 16, 0)$.

4.1.16 Since every integer is congruent mod 10 to its last digit, we need only look at the last digits of the fourth powers of the possible digits. This gives the possibilities 0, 1, 5 and 6.