MATH 406
HOMEWORK 9
DUE DECEMBER 3, 2007

(1) Simplify the following into the form $a + bi \in \mathbb{Z}[i]$.
   (a) $(2+i)^2(3+i)$
   (b) $(2-3i)^3$
(2) Determine if $\alpha \big| \beta$ in $\mathbb{Z}[i]$.
   (a) $\alpha = 2 - i$
       $\beta = 5 + 5i$
   (b) $\alpha = 1 - i$
       $\beta = 8$
(3) Let $\alpha, \beta, \gamma \in \mathbb{Z}[i]$. Prove if $\alpha \big| \beta$ and $\beta \big| \gamma$ then $\alpha \big| \gamma$.
(4) Show if $\alpha, \beta \in \mathbb{Z}[i]$ with $\alpha \big| \beta$, then $N(\alpha) \big| N(\beta)$.
(5) For each pair α, β below, find the quotient γ and the remainder ρ when α is divided by β. Verify that $N(\rho) < N(\beta)$.
   (a) $\alpha = 14 + 17i$
       $\beta = 2 + 3i$
   (b) $\alpha = 7 - 19i$
       $\beta = 3 - 4i$
(6) Show $1 + i$ divides $a + bi \in \mathbb{Z}[i]$ if and only if a and b are both even or both odd.

Date: November 28, 2007.