AMSC 714 - FALL 2022

NUMERICAL METHODS FOR STATIONARY PDEs

Instructor: RICARDO H. NOCHETTO

Time and Place: Tu-Th 5-6:15, MTH 0303

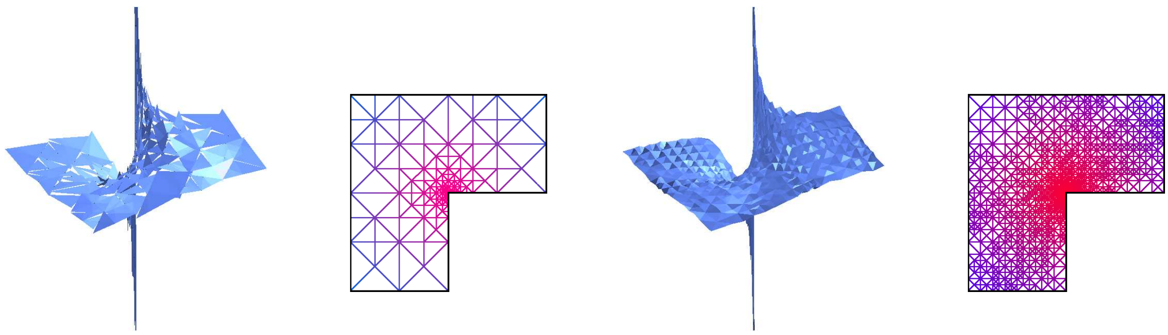
Objectives. The finite element method (FEM) is one of the most successful computational tools in dealing with partial differential equations (PDE) arising in science and engineering (solid and fluid mechanics, electromagnetism, thermodynamics, etc). The formulation of FEM, as well as finite difference methods, their properties, stability, convergence, and fast solvers (multilevel methods and preconditioners) will be discussed. Their actual implementation will also be addressed mainly via MATLAB computer projects.

Texts: [1] S.C. Brenner and L.R. Scott, *The Mathematical Theory of Finite Element Methods*, Springer (2008), 3rd edition, ISBN 978-1-4419-2611-1.

[2] D. Braess, *Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics*, Cambridge Univ. Press, (2007), 3rd edition, ISBN 978-0-521-70518-9.

Syllabus

- Maximum principle, finite difference method, error analysis.
- Variational formulation of elliptic problems and examples: the inf-sup condition.
- The finite element method and its implementation.
- Polynomial interpolation theory in Sobolev spaces.
- A priori error estimates and applications.
- A posteriori error estimates and adaptivity.
- Fast solvers: multigrid methods and multilevel preconditioners.
- Variational crimes: nonconformity, quadrature, isoparametric finite elements.
- Mixed FEM: inf-sup condition and stable spaces, applications to Stokes Flow.



Stokes flow over an L-shaped domain: Pressures and meshes for error tolerance of 5% and unstable finite element pairs (resp. DOFs): $\mathcal{P}^2$-$\mathcal{P}^1_d$ (1940), $\mathcal{P}^1$-$\mathcal{P}^1$ (4971). The oscillations do not persist under further selective refinement (nonlinear stabilization effect of adaptivity). Error estimation and adaptivity will be fully discussed in this course.

Prerequisities. Functional analysis and PDE theory (variational method, maximum principle) will be reviewed. Prior exposure to graduate level PDE and MATLAB will be useful but not mandatory. This course is an excellent complement to MATH 673 and MATH 674, which cover classical and modern PDE theory including Sobolev spaces, as well as AMSC 661, which emphasizes computational aspects of the FEM.

Evaluation: Homeworks, both theoretical (75%) and computational (25%), and a final project (to be discussed orally and play the role of last homework for the final grade). Basic MATLAB programs will be distributed and will have to be modified appropriately.