Math 808 V, Topic course: Shimura varieties

Shimura varieties play a vital role in the arithmetic aspects of the Langlands Program. This course will focus on the point of view that they are moduli spaces of abelian varieties with additional structures. Thus for most of the time we restrict our scope to the so-called PEL Shimura varieties. It will be important for us to treat these moduli spaces as schemes over a mixed characteristic base. Our aim is to survey some important arithmetic applications of these Shimura varieties after establishing their existence and basic properties.

Topics:

1. Modular curves and Siegel modular varieties over the complex numbers.
2. Siegel modular varieties as schemes: The representability of the moduli of principally polarized abelian varieties (following Mumford).
3. Shimura–Taniyama reciprocity law for CM abelian varieties; a glimpse into the theory of canonical models of Shimura varieties (preview of Prof. Rapoport's course in the fall)
4. The PEL moduli problem, representability and smoothness
5. Number of points over a finite field, and relationship to the global Langlands correspondence
6. Geometry of the mod p Shimura variety and arithmetic applications. Topics should include the supersingular loci in some low dimension Shimura varieties, Ihara's lemma, and applications such as congruences of modular forms
7. If time permits, we have the following three directions in mind for further exploration. The actual choice will depend on the amount of time left as well as the interest of the audience:
    a. bad reduction (probably an example-based survey)
    b. towards Hodge-type and abelian-type Shimura varieties
    c. relation with the geometric Satake equivalence (following Liang Xiao and Xinwen Zhu)

We plan to spend the most time on the even-numbered topics.

Prerequisites:

Basic understanding of algebraic geometry (such as Hartshorne's textbook), algebraic number theory (adeles/ideles, the statements of class field theory), and abelian varieties (such as Mumford's textbook) is assumed as prerequisites. Along the way we will encounter p-divisible groups, some p-adic Hodge theory, some etale cohomology theory, and a lot of algebraic groups. We will try to cover the minimum background as much as possible. Some prior experience with modular curves and modular forms will be helpful but not strictly needed.

For grades: If you take this course for a grade, you will need to give a presentation on an assigned topic.