Math 420, Spring 2020
First Project: Long Frontiers

presentation Thursday, 5 March, 2020
report due Friday, 13 March, 2020

This project explores the relationship between frontiers and long frontiers. Consider the following groups of assets.

(A) VFINX, VBMFX, VGSLX.
(B) from the "Projects" webpage.
(C) from the "Projects" webpage.

(1) Describe the assets including the holdings of the funds. (This information should inform some of your subsequent answers.)

(2) For each of the years ending December 31 of 2005-2019 use one-year histories with uniform weights to compute $\mathbf{m}$ and $\mathbf{V}$ for the risky assets in group (A), in groups (A) and (B) combined, and in groups (A), (B), and (C) combined.

(3) For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the minimum volatility portfolio allocation $\mathbf{f}_{\text{mv}}$. Present these in three tables (one for group (A), one for groups (A) and (B) combined, and one for groups (A), (B), and (C) combined) that lists years and the allocations for each asset rounded to the nearest thousandth. Determine if each of these portfolios is long or solvent. Comment on the implications of what you find.

(4) Assume that the safe investment for each year is the U.S. T-Bill rate available at the beginning of that year. Assume that the credit-line for each year is three points higher than the U.S. T-Bill rate. For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the tangency portfolio allocations $\mathbf{f}_{\text{st}}$ and $\mathbf{f}_{\text{ct}}$ whenever they exist. Present these in three tables as was done in part 3. Identify when each of these portfolios exists and when it does determine if it is long or solvent. Comment on the implications of what you find.

(5) For each year graph in the $\sigma\mu$-plane the three frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 and the three efficient frontiers associated with the tangency portfolios found in part 4. Comment on the implications of what you see.

(6) In a similar manner, for each year graph the efficient long frontiers, for the risky assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined, with a safe investment of U.S. T-Bills. Comment on the implications of what you see.

(7) Present one table that for each year gives the metrics

$$\omega^{\nu}_{\text{st}} = \frac{\nu_{\text{as}}}{\nu_{\text{st}}}\,, \qquad \omega^{\nu}_{\text{ct}} = \frac{\nu_{\text{as}}}{\nu_{\text{ct}}}\,,$$

for each of the three efficient frontiers graphed in part 5. Comment on the implications of what you find.

(8) Compare the efficient frontier with the efficient long frontier for each of the groupings and each of the years considered above. Determine when they intersect and give the intersection when they do. When they do not intersect determine the minimum and maximum value of $|\mathbf{f}(\mu) - \mathbf{f}_{\mathrm{l}}(\mu)|$ where $|\cdot|$ denotes the sum of the absolute values of the entries. Devise three other measures of the difference between the efficient frontier with the efficient long frontier. Present this data in a table. Comment on the implications of what you find.