Homework #1 (due September 13, 2012)

1. Let a random element X have a pdf $p(x;\theta)$ depending on a parameter $\theta \in \Theta$, $X \sim p(x;\theta)$.
A statistic $g(X)$ is called *Lehmann unbiased* estimator of a parametric function $\gamma(\theta)$ with respect to a loss function $L(g,\gamma)$ if the risk

$$R(g(X),\gamma(\theta)) = E_\theta L(g(X),\gamma(\theta)) = \int L(g(x),\gamma(\theta))p(x;\theta)dx$$

satisfies the condition

$$R(g(X),\gamma(\theta)) \le R(g(X),\gamma_1(\theta)),\ \forall \theta \in \Theta$$

for any $\gamma_1(\theta)$.
For the loss functions (i)$L(g(X),\gamma(\theta)) = (g(X)-\gamma(\theta))^2$ and (ii)$L(g(X),\gamma(\theta)) = |g(X)-\gamma(\theta)|$ find the condition of Lehmann unbiasedness of $g(X)$.

2. Let $(x_1,\dots,x_n)$ be a sample from population with pdf

$$f(x;\theta) = \theta/x^{\theta+1},\ x \ge 1$$

with $\theta > 0$ as a parameter. State the conditions on θ when the estimators are well defined.
(i) Find the method of moments $\tilde{\theta}_{MM}$ and the maximum likelihood $\tilde{\theta}_{ML}$ estimators of θ.
(ii) Show that $\tilde{\theta}_{MM} > 1$ with probability one and $\tilde{\theta}_{ML} > 0$ with probability one.
(iii) Show that $\tilde{\theta}_{MM}$ is consistent if $\theta > 1$ and $\tilde{\theta}_{ML}$ is consistent if $\theta > 0$.
(iv) Show that both $\tilde{\theta}_{MM}$ and $\tilde{\theta}_{ML}$ are positively biased, the first for $\theta > 1$, the second for $\theta > 0$.