Homework #3 (due October 4, 2012)

1. Let $\{p(x;\theta_1,\theta_2),\ (\theta_1,\ \theta_2) \in \Theta_1 \times \Theta_2\}$ be a family of positive densities of a random element $X \in (\mathcal{X},\ \mathcal{A})$.
Using the factorization theorem show that if a statistic $T_1(X)$ is sufficient for the family $\{p(x;\theta_1,\ \theta_2^*),\ \theta_1 \in \Theta_1\}$ (equivalently, sufficient for θ_1) for any given θ_2^* and $T_2(X)$ is sufficient for the family $\{p(x;\ \theta_1^*,\ \theta_2),\ \theta_2 \in \Theta_2\}$ for any given θ_1^*, then the pair $(T_1(X),\ T_2(X))$ is sufficient for $(\theta_1,\ \theta_2)$.

2. Let $x_1, \ldots, x_n)$ be a sample from a population with pdf

$$f(x;\theta) = C \exp\{-(x-\theta)^4\}$$

with θ as a parameter and C a normalizing constant.
Find the minimal sufficient statistic for θ.

3. Let $(x_1, \ldots, x_n)$ be a sample from a population with the beta distribution with parameters $\alpha > 0, \beta > 0$. Find the minimal sufficient statistic for (α, β).

4. Ch. 2, Problems 14, 28, 29.