Homework #7 (due November 29, 2012)

1. In the setup of Problems 19 (Ch. 3) and 133 (Ch. 4) the MLE of θ is $\hat{\theta}_n = M_n = \max(X_1, \ldots, X_n)$ and the UMVUE of θ is

$$\tilde{\theta}_n = M_n, \text{ if } M_n = 1, = \frac{n+1}{n} M_n, \text{ if } M_n > 1$$

(see the handout).
Calculate the $\mathrm{MSE}(\hat{\theta}_n) = E_\theta\{(\hat{\theta}_n - \theta)^2\}$ and $\mathrm{MSE}(\tilde{\theta}_n) = \mathrm{var}_\theta(\tilde{\theta}_n)$ and find the asymptotic relative efficiency of $\tilde{\theta}_n$ with respect to $\hat{\theta}_n$ defined as

$$\mathrm{raseff}_\theta(\tilde{\theta}_n; \hat{\theta}_n) = \lim_{n\to\infty} \frac{\mathrm{var}_\theta(\tilde{\theta}_n)}{\mathrm{MSE}(\hat{\theta}_n)}.$$

2. Assume that $Y_1, \ldots, Y_n$, $Y_i \sim U(0, \theta)$ are observable (and not only the X's) and let $t_n = t_n(Y_1, \ldots, Y_n)$ be the MLE of θ.
Calculate $\mathrm{var}_\theta(t_n)$ and compare it with $\mathrm{MSE}(\hat{\theta}_n)$ from #1.
Interpret the result.