**Take home exam (October 9 - 16, 2012)**

**Each problem is worth 20 points**

**1.** Let $(x_1,\ x_2)$ be a sample from a population with pdf

$$f(x;\theta) = (1/2)\exp\{-|x-\theta|\}$$

with $\theta \in \mathbb{R}$ as a parameter.
Show that the vector $(x_{1:2}, x_{2:2})$ of order statistics is the minimal sufficient statistic for $\theta$.
Try to prove the result for samples of any size.
**Hint:** Use a result proved and thoroughly discussed in class. In view of it, suffice to show that the joint pdf $p(x_1, x_2; \theta)$ is such that

$$p(x_1', x_2'; \theta)/p(x_1', x_2'; \theta_0) = p(x_1'', x_2''; \theta)/p(x_1'', x_2''; \theta_0)\ \forall\theta$$

implies $x_{1:2}' = x_{1:2}''$, $x_{2:2}' = x_{2:2}''$. You may choose $\theta_0 = 0$. Think on where to let $\theta$ run. This is the only nontrivial part.

**2.** Let $(x_1, \ldots, x_n)$ be a sample from a discrete population,

$$P(x_i = a; \theta) = \theta/6,\ P(x_i = b; \theta) = \theta/3 : P(x_i = c; \theta) = 1 - \theta/2$$

with $\theta \in (0, 1)$ as a parameter.
Find the minimal sufficient statistic for $\theta$ and investigate if it is complete.

**3.** Prove that a complete sufficient statistic is minimal.

**4.** Let $(x_1, \ldots, x_n)$ be a sample from a bivariate uniform distribution over the unit disk with center at zero and radius $\theta$, $\theta > 0$ as a parameter.
Find the minimal sufficient statistic for $\theta$ and investigate if it is complete.

**5.** Let $(x_1, \ldots, x_n)$ be a sample from a population with pdf

$$f(x;\theta) = \lambda\exp\{-\lambda(x-\theta)\},\ x > \theta$$

with $\theta \in (-\infty,\ \infty)$ as a parameter, $\lambda$ known.
(i) Find the minimal sufficient statistic and investigate if it is complete.
(ii) Construct the UMVUE of the population mean and calculate its variance.