Computer Assignment 3: MATH 340, Fall 2008

Due Wednesday, November 13

You may work alone or in teams of two people. Each team must submit a single printed solution. Solutions must contain your *relevant* MATLAB input and output (do not include commands that didn't work), and text that indicates what your commands are doing and interprets your results. (You may find one of the following commands useful in preparing your solutions: `publish`, `notebook`, or `diary`; see MATLAB's online help for details.) Organization and clarity count.

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The MATLAB command `dblquad` numerically approximates a double integral over a rectangle. Similarly, `triplequad` approximates a triple integral over a box. In this assignment, we'll use these commands on integrals for which we know the correct answer, in order to assess their accuracy. In particular, for regions of integration that are not rectangular, we'll compare two approaches for numerical integration: one is to define the function to be zero outside the region of integration (like in the definition of double and triple integrals by Riemann sums), and the other is to make a change of coordinates so that the region of integration is rectangular in the new coordinates. (Note: MATLAB R2012b and later have new functions `integral2` and `integral3` that allow integrals over non-rectangular regions, but have a more complicated syntax; please stick with `dblquad` and `triplequad` for this assignment.)

(a) Find the double integral of x^2 over the unit disk $x^2 + y^2 \leq 1$ in $\mathbb{R}^2$ by writing it as an iterated integral and using the MATLAB command `int`.

(b) Compare the exact answer with the approximate answers found by `dblquad`, first using rectangular coordinates and then using polar coordinates.

To do the integral in rectangular coordinates, use a command similar to last sample command in the the last example in the online help for `dblquad`.

Then rewrite the integral in polar coordinates and use `dblquad` to approximate that integral.

In each case, observe how long it takes the command to run and how accurate the answer is. How does the accuracy compare to the intended error tolerance of 10^{-6}?

(c) Repeat part (b), this time telling `dblquad` to use an error tolerance of 10^{-12} (see the online help for how to do this).

(d) Now use `triplequad` to approximate the volume of the unit ball $x^2 + y^2 + z^2 \leq 1$ in $\mathbb{R}^3$, using both rectangular and spherical coordinates. In each case, adjust the error tolerance (see the online help) in order to see how many digits of accuracy you can get using at most 60 seconds of computation time.

(e) Based on your results, discuss the relative merits of the two approaches to numerical integration over a nonrectangular region. Why do the two approaches produce such different results?