Computer Assignment 2: MATH 341, Spring 2014

Due Wednesday, April 9

You may work alone or in teams of two people. Each team must submit a single printed solution. Solutions must contain your *relevant* MATLAB input and output (do not include commands that didn't work), and text that indicates what your commands are doing and interprets your results. (You may find one of the following commands useful in preparing your solutions: `publish`, `notebook`, or `diary`; see MATLAB's online help for details.) Organization and clarity count.

. . . .

The differential equation

$$\theta'' + \sin\theta = 0$$

models a pendulum consisting of a rigid arm that rotates in a plane, neglecting friction and air resistance. Here θ represents the angle the arm makes from the downward vertical position, and we have chosen units of time for which the constant that multiplies $\sin\theta$ is equal to 1.

(a) Convert the second order equation to a system of two first order equations, and use `ode45` to compute numerical solutions of the system with initial conditions $\theta(0) = A$, $\theta'(0) = 0$ for various values of A: $0.1, 0.7, 1.5, 3.0$. The exact solution of this initial value problem is periodic with amplitude A. Use the numerical solutions to approximate the period of the pendulum for each of the four given amplitudes. Compare your results to the period of the linearized equation $\theta'' + \theta = 0$, and explain why some are closer than others.

(b) Here is a more analytical approach to finding the period of the pendulum. Consider more generally the equation $y'' + f(y) = 0$ for some continuous function f. Let E be an antiderivative (integral) of f. Multiply the differential equation by y', substitute $E'(y)$ for $f(y)$, and integrate with respect to t to get

$$(y')^2/2 + E(y) = C$$

where C is a constant. This equation expresses "conservation of energy", where $(y')^2/2$ corresponds to kinetic energy and $E(y)$ corresponds to potential energy. The conservation equation can be solved for y' to get a separable first order equation, which can in turn be solved for $y(t)$, at least over intervals for which y' does not change sign. Apply this approach to the pendulum equation to express the period of the oscillation with amplitude A in terms of a definite integral. Evaluate this integral using MATLAB for each of the four values of A from part (a). Compare your results to those from part (a), and discuss their accuracy.

(c) Now add damping and a periodic external force to the pendulum equation:

$$\theta'' + 0.03\,\theta' + \sin\theta = 0.5\cos\omega t.$$

The corresponding linear equation is:

$$\theta'' + 0.03\,\theta' + \theta = 0.5\cos\omega t.$$

Using initial conditions $\theta(0) = \theta'(0) = 0$, compute and graph numerical solutions for both the nonlinear and linear equations from $t = 0$ to $t = 60$ for each of the following values of the frequency ω: $0.6, 0.8, 1, 1.2$. Discuss your results in relation to the concept of resonance and the periods you found in parts (a) and (b).

(d) Three the eight numerical solutions in part (c) should reach an absolute value greater than 3. Compute and graph these solutions over longer time intervals, and form a hypothesis about which solutions remains bounded. Also, describe physically what the pendulum is doing for the solutions to the nonlinear equation.