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Problem: Estimate $\phi = 0.8$ in AR(1):

$$Z_t = 0.8 * Z_{t-1} + e_t, \quad t = 1, ..., 1000, \quad e_t \sim N(0, \sigma^2)$$

using the zero-crossings (ZC) estimator

$$\hat{\phi}_{zc} = \cos(\pi D/999)$$

where D is the number of zero-crossings observed in a plot of Z_t, $t = 1, ..., 1000$, and also by least squares (LS),

$$\hat{\phi}_{ls} = \frac{\sum_{t=2}^{1000} Z_t Z_{t-1}}{\sum_{t=2}^{1000} Z_t^2}$$

and get the mean square error (MSE) from 500 time series each of length 1000.

From ZC you get estimates: $\hat{\phi}_{zc}(1), ..., \hat{\phi}_{zc}(500)$, each approximating 0.8. From LS you get estimates: $\hat{\phi}_{ls}(1), ..., \hat{\phi}_{ls}(500)$, each approximstong 0.8

Compute two MSE's:

$$MSE(ZC) = \sum_{i=1}^{500} (\hat{\phi}_{zc}(i) - 0.8)^2/500$$

$$MSE(LS) = \sum_{i=1}^{500} (\hat{\phi}_{ls}(i) - 0.8)^2/500$$

See which estimator gives a smaller MSE.

Use SAS code with do-loops as needed.