STAT730 Homework

1. Appreciation of dependence.

Assume $X_1, .., X_n$ are observations from a $0-1$ stationary Markov chain, with parametrs p, λ_1:

$$P(X_t = 1) = p, \quad P(X_t = 1 | X_{t-1} = 1) = \lambda_1$$

Let

$$S_n = X_1 + X_2 + \cdots + X_n$$

Obtain:

a. $P(S_n = k)$.

b. Compare (a) with the binomial distribution $b(n, p)$.

2. Smoothing.

Consider the sinusoid plus normal noise:

$$y_t = \cos(0.05t) + u_t, \quad t = 1, 2, ..., 100$$

where u_t are iid $N(0, 0.25^2)$. Smooth the curve using:

1. Moving average (q=0, m=3)
2. Polynomial smoothing (q=2,m2)
3. Polynomial smoothing (q=2,m=3)
4. Smoothing spline.
5. Lowess.
6. Running medians (m=3).

3. Random telegraph.

Consider a process $\{X(t), t \in [0, \infty)\}$ such thst

$$P(X(t) = 1) = P(X(t) = -1) = 1/2$$

and where the times at which sign changes occur constitute a Poisson process $\{N(t), t \geq 0\}$ with parametre λ, and

$$p(k) = P(N(t) = k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

Show that $X(t)$ is weekly stationary with autocovariance $R(u) = e^{-2\lambda|u|}$. Recall $N(t)$ has independent incremenrs.

4. AR(1).
Condider the AR(1) process with parametr ϕ,

$$z_t = \phi z_{t-1} + \epsilon_t, \quad t = 1, 2, ...$$

where ϵ_t is white noise. Is z_t stationary
a. if $|\phi| < 1$?
b. if $|\phi| > 1$?

5. MA Representation.
Assume $\{X_t,\ t = 0, \pm 1, ...\}, t = 0, \pm 1, ...$ is weekly stationary with mean $EX_t = 0$, and a continuous spectral density $f(\lambda) > 0$. Show that X_t admits the MA representation,

$$X_t = \sum_r \gamma_r v_{t-r}$$

where $\{v_r\}$ is white noise.
Hint: Find an appropriate representation for v_t.

6. AR(1) Estimation.
Consider an AR(1) time series

$$z_t = \phi z_{t-1} + \epsilon_t, \quad , t = 1, 2, ...N$$

where $|\phi| < 1$, and ϵ_t are iid N(0,σ^2).

Compare by a simulation the least squares estimate of ϕ with the zero-crossing estimate.

7. Cosine Formula
Consider a zero-mean stationary Gaussian time series. Prove the cosine formula,

$$\rho_1 = \cos\left(\frac{\pi E(D_1)}{N-1}\right)$$

8. Coherence, Lagged Coherence, Residual Coherence
Get any two financial time series of your choice and evaluate the coherence, lagged coherence, and the residual coherence by a bar plot. State your conclusions.