Math 136 — Differential geometry **Due**
Problem Set 6 **October 22**

Problem 1. Consider the *torus of revolution* $T_{a,b}$ obtained by rotating the circle $\gamma(u) = (a + b\cos\frac{u}{b}, b\sin\frac{u}{b})$ about the z axis. It has parametrization

$$f(u,v) = \Big((a + b\cos\frac{u}{b})\cos v, (a + b\cos\frac{u}{b})\sin v, b\sin\frac{u}{b}\Big)$$

where $0 < u < 2\pi b$ and $0 < v < 2\pi$ and $a > b > 0$. Note that γ is an arc-length parametrization.

(1) Compute the first fundamental form of $T_{a,b}$.
(2) Compute the Gauss curvature K and the mean curvature H of $T_{a,b}$.
(3) Compute the integral

$$W(a,b) = \int_{T_{a,b}} H^2 dA$$

as a function of (a,b) and show that $W(a,b) \geq 2\pi^2$ with equality achieved when $a/b = \sqrt{2}$.

Problem 2. Consider the surface S described by the equation $z = xy$. Determine the lines of curvature and the asymptotic curves of S.