Math 406 — EXAMINATION 1 — 27 February 2009

Work each of the 5 numbered problems on a separate answer sheet. Each answer sheet must have your name and the problem number (= page number). Explain your solutions clearly. Your solutions must include enough detail to justify your conclusions. No calculators allowed. Good luck.

1. [15] (a) Use induction to show that $2^n < n!$ for all $n \geq 4$.

[10] (b) Show that $3|(f_n + f_{n+4})$ for all positive integers n.

2. [15] (a) Assume that $(a, b) = 1$. Show that $(a + 2b, 2a + b)$ is either 1 or 3.

[10] (b) Assume that $a^2|n^2$. Show that $a|n$.

3. [20] Use the Euclidean algorithm to find the greatest common divisor of 160 and 36 and express it as a linear combination of 160 and 36.

4. [15] Find all integer solutions to $15x + 9y = 57$.

5. [15] Show that if a is odd then $a^2 \equiv 1 \pmod 8$.