Math 406 — EXAMINATION 2 — 6 April 2009

Work each of the 6 numbered problems on a separate answer sheet. Each answer sheet must have your name and the problem number. Explain your solutions clearly. Your solutions must include enough detail to justify your conclusions. No calculators allowed. Good luck.

1. [15] Find all incongruent solutions to $20x \equiv 12 \pmod{36}$. Your answers should be least non-negative residues.

2. [15] Find the smallest positive integer which is divisible by 7, leaves a remainder of 3 when divided by 5, and leaves a remainder of 2 when divided by 8. [No credit for guessing or random search!]

3. [15] Find the remainder when 25! is divided by 29.

4. [10] (a) Assume that $(a, 12) = 1$. Show that $a^6 \equiv a^2 \pmod{12}$.

[10] (b) Assume that $(a, 60) = 1$. Show that $a^6 \equiv a^2 \pmod{60}$.

5. [15] Show that $\phi(3n) = 2\phi(n)$ if $3 \not| n$ but that $\phi(3n) = 3\phi(n)$ if $3|n$.

6. [10] (a) Prove that if $(2^m - 1)$ is prime then $2^{m-1}(2^m - 1)$ is perfect.

[10] (b) Let m be a perfect number and let $k > 1$. Show that $\sigma(km) > 2km$.