Math 406 — FINAL EXAMINATION — 15 May 2009

Work each of the 10 numbered problems on a separate answer sheet. Each answer sheet must have your name and the problem number. Explain your solutions clearly. Your solutions must include enough detail to justify your conclusions. No calculators allowed. Good luck.

1. [20] Use the Euclidean algorithm to find $(150, 42)$ and to express it as a linear combination of 150 and 42.

2. [15] Assume that $(a, b) = 1$. Show that $(a + b, a - b)$ is either 1 or 2.

3. [15] Assume that $a^3|b^2$. Show that $a|b$.

4. [20] Use induction to show that $5^n \equiv 1 + 4n \pmod{16}$ for all $n \geq 1$.

5. [15] Find all incongruent solutions to $15x \equiv 6 \pmod{48}$. Your answers should be least non-negative residues.

6. [15] (a) Find the remainder when 38! is divided by 41.

[15] (b) Use Euler's Theorem to find an inverse of 5 modulo 18 — express your answer as the least positive residue.

7. [15] Show directly that 36 has no primitive roots — you will receive no credit for quoting the theorem characterizing such integers or for using brute force.

8. We know that 5 is a primitive root of 17. Below is a table of indices to base 5. Use indices to find all solutions to the following congruences. Carefully state the moduli of your answers, which should be least positive residues.

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$ind_5 x$	16	6	13	12	1	3	15	2	10	7	11	9	4	5	14	8

(1)

[15] (a) $x^4 \equiv 4 \pmod{17}$

[15] (b) $6^x \equiv 7 \pmod{17}$

9. [20] Assume that p and q are odd primes and $p \equiv q \pmod{28}$. Show that $\left(\frac{7}{p}\right) = \left(\frac{7}{q}\right)$.

10. [20] Evaluate $\left(\frac{140}{239}\right)$. Explain carefully. Note that 239 is prime.