MATH 406 – HOMEWORK I

(due Monday 6 February 2012)

1. Assume that $a, b \in \mathbb{Z}$ and $b \neq 0$. Let $S = \{a + bk \mid k \in \mathbb{Z},\ a + bk > 0\}$. Show that S has a smallest element but no largest element.

2. Use Dirichlet Approximation to find integers a, b with $1 \leq a \leq 6$ such that $|a\sqrt{5} - b| < 1/6$. Show your work!

3. Prove that $[x] + [x + \frac{1}{2}] = [2x]$ for all real numbers x.

4. Use mathematical induction to prove that $n < 2^n$ for all positive integers n.

5. Use (strong) mathematical induction to show that every integer $n \geq 8$ can be written as $n = 3a + 5b$ for non-negative integers a, b.

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.